Let \( f(z) \frac{d^n}{dz^n}(1/f(z)) = P_n(z) \) be a monic polynomial having \( n \) distinct real roots, for \( n = 0, 1, \ldots \). Assume also that \( f(\pm \infty) = \infty \). Note that \( f(z) \) is an entire function of finite type.

**Claim:** For \( q \) and \( m \) nonnegative integers,

\[
P(z) = \sum_{\gamma} \binom{q}{\gamma} \frac{m!}{(m-\gamma)!} z^{m-\gamma} P_{q-\gamma}(z),
\]

has \( q + m \) distinct real roots if \( q > m \), and \( 2q \) distinct nonzero real roots if \( q \leq m \).

**Remark:** This generalizes Problem 5681 of the AMM [P], where this was proposed for \( P_n(z) = H_n(z) \), the Hermite polynomials.

**Proof:** After rewriting the sum in (1),

\[
P(z) = f(z) \frac{d^q}{dz^q}(z^m/f(z)),
\]

using \( z^m/f(z) \to 0 \) when \( z \to \pm \infty \), Rolle’s theorem and induction on \( q \), the claim follows. \( \square \)

**Reference:**