ADDENDUM TO: ALGEBRAIC SCHWARZ METHODS FOR THE NUMERICAL SOLUTION OF MARKOV CHAINS*

IVO MAREK† AND DANIEL B. SZYLD‡

In this addendum, we expand a portion of the proof of Theorem 4.1 of [1]

We show now that $\mathcal{N}(I-T) = \mathcal{N}(A)$. According to the construction of T, $\mathcal{N}(A) \subseteq \mathcal{N}(I-T)$. Thus, we need to show now that there is no element in $\mathcal{N}(I-T)$ which does not belong to $\mathcal{N}(A)$. Let us assume, on the contrary that such element exists. Any element of $z \in \mathcal{N}(I-T)$ which does not belong to $\mathcal{N}(A)$ has to have a form $z = u + y$, with $u \in \mathcal{N}(A)$, $y \in \mathcal{R}(A)$.

By construction $y \neq 0$, otherwise $z = u \in \mathcal{N}(A)$, and we are assuming that $z \notin \mathcal{N}(A)$.

Since $\mathcal{N}(A) \subseteq \mathcal{N}(I-T)$, we have that $Tu = u$, and thus, we also have that $Ty = y$. This implies that $\lim_k T^k y = Qy = y$, and thus $y \geq 0$.

In summary, we have $y = Ax$ for some x and $y \geq 0$, with $y \neq 0$.

On the other hand $y^T e = x^T A^T e = 0$, a contradiction.

REFERENCES

[1] Ivo Marek and Daniel B. Szyld Algebraic Schwarz Methods for the Numerical Solution of Markov Chains. *Linear Algebra and its Applications*, **386** (2004) 67–81.

*This version 4 December 2005

†Czech Institute of Technology, School of Civil Engineering, Thakurova 7, 166 29 Praha 6, Czech Republic (marek@ms.mff.cuni.cz).

‡Department of Mathematics, Temple University (038-16), 1805 N. Broad Street, Philadelphia, Pennsylvania 19122-6094, USA (szyld@math.temple.edu).