1. Consider the $2 \times 3$ matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$  

Show an example of a matrix $X$ so that $AX = I$, but such that $XA \neq I$ (show the two products explicitly); see last sentence in book in page 117.

2. Let $A \sim B$, i.e., there exist nonsingular matrices $P, Q$ such that $B = PAQ$.
   (a) Show that if $B$ is singular, then $A$ is singular.
   (b) Show that if $A$ is singular, then $B$ is singular.

3. Let

$$A = \begin{bmatrix} 4 & -8 & 5 \\ 4 & -7 & 4 \\ 3 & -4 & 2 \end{bmatrix}.$$  

(a) Compute the LU factorization of $A$.
(b) Let $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Find the solution to $Ax = b$ using the LU factorization from (a).
(c) Check that $L^{-1}A = U$.

4. (a) Give an example of a $2 \times 2$ matrix $A$ which cannot be written as $A = LU$, but which it can be permuted so that there exists an LU factorization of the permuted matrix $PA = LU$.
(b) Give an example of a $2 \times 2$ matrix $A$ with $A_{22} = 0$ for which an LU factorization exists and the matrix $U$ has a nonzero entry in the (2,2) position.