

## Problem Set 3

(Out Tue 02/15/2022, Due Thu 02/24/2022)

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**Problem 4**

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Download the Matlab file `mit18086_fd_transport_limiter.m` from the course website. Apply the program (with  $k = 0.9h$ ) to the linear advection equation

$$\begin{cases} u_t + u_x = 0 \\ u(x, 0) = u_0(x) \end{cases}$$

on  $x \in [0, 1]$  with periodic boundary conditions, and  $t \in [0, 1]$ , with the following sets of initial conditions:

(A)  $u(x, 0) = \sin(2\pi x)$

(B)  $u(x, 0) = \begin{cases} x & x \in [0, \frac{1}{2}) \\ 1 - x & x \in [\frac{1}{2}, 1) \end{cases}$

(C)  $u(x, 0) = \begin{cases} 1 & x \in [0, \frac{1}{2}) \\ 0 & x \in [\frac{1}{2}, 1) \end{cases}$

a) As in Problem 1 (problem set 1), conduct a numerical error analysis (in  $\|\cdot\|_\infty$  and  $\|\cdot\|_1$ ) of the method with the following schemes/limiters: (i) none (upwind); (ii) none (Lax-Wendroff), (iii) superbee, (iv) van-Leer, (v) minmod [you need to add minmod to the program]. Explain your observations.

b) Now change the final time to  $t = 10$ , and re-do the convergence study. Explain your observations.