Temple 3044 Numerical Analysis II Spring 2012

Problem Set 4

(Out Thu 02/09/2012, Due Thu 02/16/2012)

Instructions

- Problems marked with **(T)** are theory problems. Their solutions are to be submitted on paper.
- Problems marked with **(P)** are practical problems, and require the use of the computer. Their solutions are to be submitted on paper, and usually require two parts: (a) a description of the underlying theory; and (b) code segments, printouts of program outputs, plots, and whatever it required to convince the grader that you have understood the theory and addressed all practical challenges appropriately.

Generally, naked numbers are not acceptable. Solutions must include a short write-up describing the problem, your solution technique, and procedural details. To include a computer printout use the cut and paste method for placement of materials in your work. All things must be clearly labeled.

Problem D

(P) We would now like to use our beloved Lotka-Volterra predator-prey model

$$\frac{d}{dt}\vec{x}(t) = \vec{f}(\vec{x}(t)) \ , \tag{1}$$

where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\vec{f}(\vec{x}) = \begin{pmatrix} f_1(x,y) \\ f_2(x,y) \end{pmatrix} = \begin{pmatrix} x - 4xy \\ -y + 5xy \end{pmatrix}$, to perform a numerical error analysis of various numerical methods. To that end, consider $t \in [0, 31]$, and initial conditions $\vec{x}(0) = \begin{pmatrix} 0.73 \\ 0.25 \end{pmatrix}$.

(a) First, obtain a reference solution[1] for the final state $\vec{x}(31)$, by using RK4 with step size $h = 10^{-4}$. Call this vector $\vec{x}_{\text{true}}$.

(b) For a sequence of time steps $h = 10^\beta$, where $\beta \in \{-3.0, -2.75, -2.5, -2.25, -2.0, -1.75, -1.5, -1.25, -1.0\}$, approximate the final state $\vec{x}(31)$ by using the following numerical schemes: (i) forward Euler, (ii) RK4, and (iii) Crank-Nicolson.[2]

(c) For each of the 27 results for the final state that you computed in (b), compute the error $\|\vec{x}(31) - \vec{x}_{\text{true}}\|_2$, and plot the sequence (with respect to h) of errors in three curves in log-log scale. Explain what the slopes of these curves tell you.

(d) In two new figures, plot the approximate solution curves $(x(t), y(t))$, obtained with (i) RK4 and (ii) Crank-Nicolson for system (1), with $t \in [0, 30000]$ and time step size $h = 0.3$.[3] What do you observe? Try to explain your observations.

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[1] If an analytical solution is unavailable, one can always use a highly resolved computational solution as a reference.

[2] Use three Newton iteration steps per time step.

[3] Yes, you are doing a lot of time steps here. See this as a simulation of an ecological systems over a millennium time scale.