Temple 8024 **Numerical Differential Equations II** **Fall 2010**

Problem Set 4

(Out Thu 10/28/2010, Due Thu 11/11/2010)

Problem 9

The fire control of New South Wales would like to test a new approach to impede the propagation of bush fires: In a checkerboard pattern, regular squares of $1\,\text{km} \times 1\,\text{km}$ are sprayed so that the propagation speed of the fire front is slowed down.

(1) Write a program that simulates the advance of a fire front that starts in the center of an untreated square and moves outward in its normal direction, with a velocity that is $1\,\text{km/h}$ in the untreated squares, and $\varepsilon\,\text{km/h}$ in the sprayed squares.

(2) Create a function $d(\varepsilon)$, where d denotes the largest distance of the fire from the origin at the final time $T_{\text{final}} = 24\,\text{h}$. Do so by running your simulation for a whole range of values $\varepsilon \in [\frac{1}{100}, 1]$. Also plot the shape of the burning region for $\varepsilon \in \{\frac{1}{100}, \frac{1}{3}, \frac{4}{5}, 1\}$.

(3) Explain your results: Are there any critical values of ε at which a transition in the fire shape occurs? Is the idea of a checkerboard spraying a good one?

Problem 10

Consider the Korteweg–de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0 \text{ on } x \in [-1, 1[\tag{1}$$

with periodic boundary conditions.

Write a highly accurate spectral code for this equation, and demonstrate its efficiency by running the initial conditions $u_0(x) = f_{400}(x + 0.7) + f_{200}(x)$ up to time $t = 0.1$ (or even further).

Compare this new code with the code that you developed in Problem 8. How much faster is it to achieve the same accuracy?