

# Quasi-particle excitations in a ferromagnet, near a quantum critical point

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## Abstract

The quasi-particle excitations of a ferromagnet, near a quantum critical point are examined. The weak ferromagnetic state is described by the Hubbard model. The effects of emission or absorption of Goldstone modes on the quasi-particles is examined, as are the effects of the incoherent spin-flip excitations. It is found that, unlike the strong ferromagnet, the Goldstone modes have negligibly small effects on the quasi-particle spectrum and the strongest effects originate from the incoherent spin-flips. The incoherent spin excitations produce large quasi-particle mass enhancements and also produce an incoherent peak in both the off Fermi energy majority and minority spin spectral density.

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The effects of incoherent magnetic fluctuations on the quasi-particle spectrum on the paramagnetic side of a quantum critical point have been thoroughly investigated [1–3], based on the framework of the Stoner–Wohlfarth mean-field model of ferromagnetism [4,5]. It was found that the emission and absorption of damped spin-waves can produce large quasi-particle mass enhancements and can result in large enhancements of the linear  $T$  term in the low-temperature electronic

specific heat. Recent angle resolved photoemission experiments [6] have revealed the quasi-particle dispersion relation in ferromagnetic Fe, and have attributed the mass renormalization to emission and absorption of coherent spin waves. Here, we examine the effect of both coherent and incoherent spin fluctuations on the quasi-particle excitation spectrum.

The transverse dynamic susceptibility, is calculated within the RPA [7], and has the form

$$\chi^{+-}(v, q) = \frac{\chi_0^{+-}(v, q)}{1 - U\chi_0^{+-}(v, q)}, \quad (1)$$

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where  $\chi_0^{+-}(v, q)$  is the mean-field [4,5] susceptibility. In the ferromagnetic state, the mean-field susceptibility is given by

$$\chi_0^{+-}(v, q) = \frac{1}{N} \sum_k \left( \frac{f_{\underline{k}\uparrow} - f_{\underline{k}+q\downarrow}}{\Delta + \varepsilon_{\underline{k}+q} - \varepsilon_{\underline{k}} - \hbar v} \right), \quad (2)$$

where  $\Delta$  is the mean-field exchange splitting. For small  $q$ , the Goldstone modes [8] are found to obey the dispersion relation

$$\hbar v = Dq^2, \quad (3)$$

where the spin-wave stiffness constant  $D$  is given by the expression [7],

$$D = \frac{U}{3N\Delta} \sum_k \left[ \left( \frac{f_{k\uparrow} + f_{k\downarrow}}{2} \right) \nabla^2 \varepsilon_k - \left( \frac{f_{k\uparrow} - f_{k\downarrow}}{\Delta} \right) |\nabla \varepsilon_k|^2 \right]. \quad (4)$$

For weak ferromagnets, the Goldstone mode spectrum only exist below a critical value of  $q$ , given by  $q_c = k_{F\uparrow} - k_{F\downarrow}$ . For larger  $q$ , the Goldstone spectrum merges with the Stoner continuum and is rapidly attenuated. The electronic self-energy due to the emission and absorption of spin-flip excitations is given by the expression

$$\Sigma_\sigma(\omega, k) = \frac{U^2}{N} \sum_q \int_{-\infty}^{\infty} \frac{\hbar dv}{\pi} \text{Im} \chi^{-\sigma\sigma}(v + i\eta, q) \times \left[ \frac{1 - f_{\underline{k}-q-\sigma} + N(\hbar v)}{\hbar\omega - \varepsilon_{\underline{k}-q} - \sigma\Delta/2 + \mu - \hbar v} \right]. \quad (5)$$

Near the quantum critical point, where the Goldstone mode only exists for  $q$  below  $q_c$ , the real part of the self-energy due to the Goldstone modes is of the order of  $\Delta(q_c/k_F)^3$ . This contribution to the self-energy is extremely small, highly  $k$  dependent, and shows a marked frequency asymmetry about  $\omega = 0$ . By contrast, the self-energy due to incoherent spin-flips is relatively large and  $k$  independent and is non-zero on both sides of the Fermi energy. The rapid  $\omega$  variation of the total self-energy is shown in Fig. 1, which is calculated for  $M = \frac{1}{4}$ . The linear  $\omega$  dependence of the real part of  $\Sigma_\sigma(\omega, k)$  near  $\omega =$

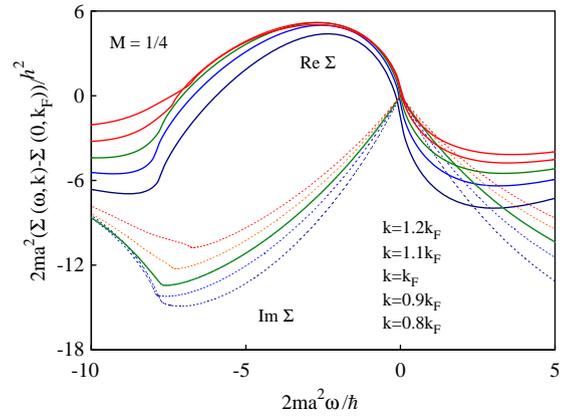


Fig. 1. The frequency dependence of the real (solid-line) and imaginary (broken-line) parts of the majority spin-self energy, for various values of  $k$ .

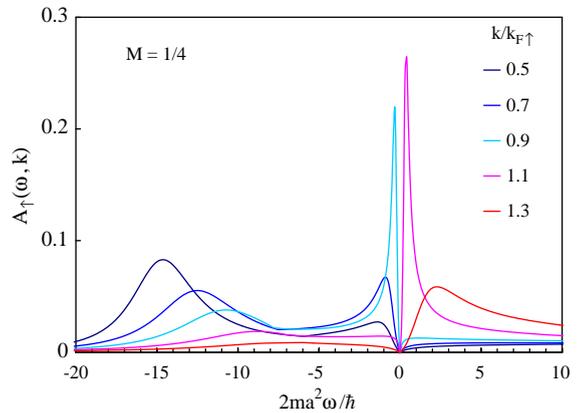


Fig. 2. The up-spin electronic spectral density  $A_\uparrow(\omega, k)$  as a function of frequency, for various values of  $k$ . The quasi-particle peaks are located near the Fermi energy ( $\omega = 0$ ), and the incoherent peaks are located below the lower edge of the mean-field up-spin sub-band.

0 produces a quasi-particle mass enhancement of the order of 8. The electronic spectral density, defined by

$$A_\sigma(\omega, k) = -\frac{1}{\pi} \text{Im} G_\sigma(\omega + i\eta, k) \quad (6)$$

is shown in Fig. 2. It is seen that in addition to the narrow quasi-particle peaks located near the Fermi energy, the spectra has weakly dispersive

incoherent peaks located at energies below the bottom of the spin-split mean-field band. Although these incoherent peaks are not true bound states, they are related to the two-hole bound states predicted for the minority spin subbands of strong ferromagnets [9–12].

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