Corrections and Updates for
“Multiplicative Invariant Theory”
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page 18, line 6: \( G \) and \( H \) are interchanged. It should read “Then the \( G \)-module \( \mathbb{Z}[G] \otimes_{\mathbb{Z}[H]} M \) is in fact a \( G \)-lattice; . . .” (Thanks to Robert Fossum.)

page 21, line -1: A backslash is missing in front of “emph”. It should be “The group \( G \) is called \( k \)-reflection group on \( V \) if \( G = R^k(H) \).” (Thanks to Robert Fossum.)

page 60, Table 3.1: The Hironaka decomposition of the invariant algebra \( \mathbb{Z}[L]^G \) for the group \( G \cong S_3 \) (third group in table) is missing the summand \( \mu_2 \mathbb{Z}[\mu_1, \mu_2] \). The correct entry in this row should be: “semigroup algebra \( \mathbb{Z}[\mu_1, \mu_2] \oplus \mu_3 \mathbb{Z}[\mu_1, \mu_2] \oplus \mu_2 \mathbb{Z}[\mu_1, \mu_2] \)”. 

page 103, line -1: “\( k \)-reflection” should be replaced by “bireflection”; so 
\[
\mathcal{R}^2(\mathcal{H}) = \langle g \in \mathcal{H} \mid g \text{ acts as a bireflection on } L \rangle.
\]

page 126, line 8 and page 127, line 4: The list of “primes” \( p = 47, 112, 223, \ldots \) is wrong as stated; it should be \( p = 47, 113, 233, \ldots \). Thus, on page 127, it should read “In particular, the aforementioned non-rational extensions \( \mathbb{Q}(\text{Cl}_p)/\mathbb{Q} \) (\( p = 47, 113, 233, \ldots \)) are not stably rational either.” (Thanks to Don Passman.)

page 126, lines -9 and -12: \( K \) should be replaced by \( k \) twice: it should read “ . . . Saltman [175] for infinite \( k \ldots \) and “ . . . \( E/F \) of \( k \) with group \( G \ldots \)”.

page 149, line -4: The answer to Problem 2, when stated more generally for \( k \)-reflections, is definitely negative. Alex Zaleskii has shown me the following example. The simple group \( \text{SL}_2(\mathbb{F}_{32}) \) has an irreducible Steinberg module, \( V \), of dimension 32 over \( \mathbb{Q} \). The restriction of \( V \) to the Sylow 2-subgroup of \( \text{SL}_2(\mathbb{F}_{32}) \) is the regular module. Therefore, each involution of \( \text{SL}_2(\mathbb{F}_{32}) \) acts as a 16-reflection. Furthermore, the involutions generate \( \text{SL}_2(\mathbb{F}_{32}) \). On the other hand, if \( g \in \text{SL}_2(\mathbb{F}_{32}) \) is an element of order 31, then its fixed points on \( V \) have dimension 2 and \( \langle g \rangle \) is an isotropy group on \( V \). Thus, this isotropy group is only generated by a 30-reflection.

page 150, line 12: Omit “over \( \overline{\mathbb{Z}} \)”.

page 154, line 5 after Problem 7: Replace \(|G|/r\) by \( r/|G| \) (twice).
Problem 14 (page 159): This has been solved in the affirmative; the reference is:
Now all multiplicative invariant fields of transcendence degree at most 3 are known to be rational over the base field.

Example 8.11.3 (page 122); see also Problem 4 (page 150/1): The Cohen-Macaulay problem for “vector invariants” is resolved (in the positive): the Cohen-Macaulay property follows as a special case of Theorem 1.2 in Blum-Smith and Marques, *When are permutation invariants Cohen-Macaulay over all fields?*, arXiv:1802.06735.