Math 127 Review – Test 2

Chapter 14 Review

35. If \( w = \sqrt{x + y^2}/z \), where \( x = e^{2t} \), \( y = t^3 + 4t \), and \( z = t^2 - 4 \), use the Chain Rule to find \( dw/dt \).

36. If \( z = \cos xy + y \cos x \), where \( x = u^2 + v \) and \( y = u - v^2 \), use the Chain Rule to find \( \partial z/\partial u \) and \( \partial z/\partial v \).

40. The length \( x \) of a side of a triangle is increasing at a rate of 3 in/s, the length \( y \) of another side is decreasing at a rate of 2 in/s, and the contained angle \( \theta \) is increasing at a rate of 0.05 radian/s. How fast is the area of the triangle changing when \( x = 40 \) in, \( y = 50 \) in, and \( \theta = \pi/6 \)?

46. Find the directional derivative of \( f(x, y, z) = x^2y + x\sqrt{1 + z} \) at the point \((1, 2, 3)\) in the direction of \( \mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \).

48. Find the direction in which \( f(x, y, z) = ze^{xy} \) increases most rapidly at the point \((0, 1, 2)\). What is the maximum rate of increase?

54. Find the local maximum and minimum values and saddle points of the function \( f(x, y) = (x^2 + y)e^{y/2} \).

15.3: 37 Sketch the region of integration and change the order of integration.
\[
\int_0^4 \int_0^{\sqrt{x}} f(x, y) \, dy \, dx
\]

15.8: 29 (mod.) Find the volume of the solid \( E \) that lies above the cone \( z = \sqrt{x^2 + y^2} \) and below the sphere \( x^2 + y^2 + z^2 = 1 \).

Chapter 15 Review

13. Calculate the iterated integral by first reversing the order of integration.
\[
\int_0^1 \int_x^1 \cos(y^2) \, dy \, dx
\]

22. Evaluate \( \iint_D x \, dA \), where \( D \) is the region in the first quadrant that lies between the circles \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 2 \).

27. Evaluate \( \iiint_E yz \, dV \), where \( E \) lies above the plane \( z = 0 \), below the plane \( z = y \), and inside the cylinder \( x^2 + y^2 = 4 \).

30. Find the volume of the solid under the surface \( z = x^2y \) and above the triangle in the \( xy \)-plane with vertices \((1, 0), (2, 1), \) and \((4, 0)\).
16.1: 24 Find the gradient vector field of \( f(x, y, z) = x \cos(y/z) \).

16.2: 22 Evaluate the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( \mathbf{F}(x, y, z) = z \mathbf{i} + y \mathbf{j} - x \mathbf{k} \) and \( C \) is given by the vector function \( \mathbf{r}(t) = t \mathbf{i} + \sin t \mathbf{j} + \cos t \mathbf{k}, \ 0 \leq t \leq \pi \).

Chapter 16 Review

6. Evaluate the line integral \( \int_C \sqrt{xy} \, dx + e^y \, dy + xz \, dz \), where \( C \) is given by \( \mathbf{r}(t) = t^4 \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}, \ 0 \leq t \leq 1 \).