Math 8062 Homework 3

Due Thursday, 2/17/11

1. There is a standard way to glue together two (connected) manifolds M and N of the same dimension. Remove an open ball B^n from each of M and N, and glue $M \setminus B^n$ to $N \setminus B^n$ along the two $(n-1)$ dimensional boundary spheres. The resulting manifold is called the *connected sum* of M and N, and is denoted $M \# N$.

a) Prove that when $n \geq 3$, $\pi_1(M \setminus B^n) \cong \pi_1(M)$.

b) Prove that when $n \geq 3$, $\pi_1(M \# N) \cong \pi_1(M) * \pi_1(N)$.

2. Let M be a manifold of dimension $n \geq 4$, and let C be a knot (i.e., an embedded circle) in M.

a) Prove that $\pi_1(M \setminus C) \cong \pi_1(M)$.

b) Is the hypothesis that $n \geq 4$ necessary?

3. Let $X = S^2 \cup A$, where A is an axis connecting the north and south poles of S^2. Find a cell complex structure on X, and use it to compute the fundamental group.

4. Do problem 8 on page 53 of Hatcher.

5. Do problem 14 on page 54 of Hatcher.

General hint: for some of these problems, van Kampen's theorem will be more useful than Proposition 1.26 (the application to cell complexes).