TEMPLE UNIVERSITY

Department of Mathematics

Applied Mathematics and Scientific Computing Seminar

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Ginzburg-Landau minimizers with prescribed degrees. Emergence of vortices and existence/nonexistence of the minimizers

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Let $\Omega \subset \mathbf{R}^2$ be a domain with a hole ω. In the domain $A = \Omega \setminus \omega$ consider a class $\mathcal{J}$ of complex valued maps having degrees 1 and 1 on $\partial\Omega$, $\partial\omega$ respectively.

In a joint work with P. Mironescu we show that if $\mathrm{cap}(A) \geq \pi$ (subcritical domain), minimizers of the Ginzburg-Landau energy

$$E_\kappa(u) = \frac{1}{2}\int_\Omega \left(|\nabla u|^2 + \frac{1}{2\kappa^2}(|u|^2 - 1)^2 \right) dx$$

exist for each κ. They are vortexless and converge in $H^1(A)$ to a minimizing S^1-valued harmonic map as $\kappa \to 0$. When $\mathrm{cap}(A) < \pi$ (supercritical domain), for small κ, we prove that the minimizing sequences/minimizers develop exactly two vortices—a vortex of degree 1 near $\partial\Omega$ and a vortex of degree -1 near $\partial\omega$. It was conjectured that the global minimizers do not exist for small κ.

In a subsequent work with D. Golovaty and V. Rybalko this conjecture was proved. It was shown that, when $\mathrm{cap}(A) < \pi$, there exists a finite threshold value κ_1 of the Ginzburg-Landau parameter such that the minimum of E_κ is not attained in $\mathcal{J}$ when $\kappa > \kappa_1$, while it is attained when $\kappa < \kappa_1$. No standard elliptic estimates worked here and our proof is based on an introduction of an auxiliary linear problem which allows for a sufficiently tight explicit energy estimate.