The heat trace, zeta-function, and resolvent of elliptic operators on conic manifolds

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Abstract: On smooth compact manifolds (with and without boundary), the heat equation method is at the core of many investigations in spectral geometry, index theory, and topology. In the late 1970's, Cheeger initiated such investigations on manifolds with geometric singularities. Probably the simplest singularities to consider are conical singularities. Since the geometry is incomplete the Laplacian is typically not essentially selfadjoint, and 'boundary conditions' associated with the singular locus are to be taken into account. In his seminal paper, Cheeger studied the heat kernel of the Friedrichs Laplacian on manifolds with cone-like singularities.

The topic has been taken up again in the late 1980's and 1990's, also motivated by applications in mathematical physics. More recently, again driven by physicists, the question arose whether it was possible to understand the heat kernel of generic selfadjoint extensions of elliptic operators on conic manifolds. In 2008, Kirsten, Loya, and Park studied the Laplacian under strong assumptions on the geometry near the singularities and showed that the heat kernel and zeta-function exhibit in general rather unexpected 'exotic' effects.

In this talk I plan to report on joint work with J. Gil and G. Mendoza where we were able to obtain a completely general structural result about the resolvent, heat kernel, and zeta-function of elliptic operators on conic manifolds. Time permitting, I will also discuss extensions of this result to more complicated geometric singularities.