

A Biography of James Waddell Alexander

There are so few records available concerning James Waddell Alexander that one of the foremost topologists of the first half of the twentieth century is in danger of being forgotten. Even the records that are readily available are insubstantial.

Alexander came from a well-to-do family. His great-great-grandfather, the first professor and principal of the Princeton Theological Seminary, was so respected that a street in the town of Princeton and two buildings on campus are named for him. His great-grandfather was rector of a congregation that happened to include the founder of the Equitable Insurance Co., so several members of his family rose to high positions within the firm. Alexander's grandfather was one of several relatives who became presidents or vice-presidents of the firm. Alexander's father was a successful and well-known painter who knew Claude Debussy, Henry James, and Auguste Rodin, among others.

Alexander himself was educated in Paris and New York before attending Princeton, where he graduated in 1910 with a degree in mathematics. He was known then for his radical political views and his tendency to participate in marches and speak on soapboxes. Alexander then studied for under Oswald Veblen, another famous topologist, visited the Universities of Paris and Bologna in 1912, and returned to America to finish his Ph.D. in 1915. His thesis, which was published later that year in the *Annals of Mathematics*, was not written on topology because it was thought to be too risky.

Alexander then started his career in mathematics at Princeton. His first achievement in topology was to provide a basis for the contributions of Poincaré. Next, he established the topological invariance of the Betti numbers and torsion coefficients, and while he only dealt with the three-dimensional case, his arguments could be extended easily to the general case. In 1918, he temporarily gave up his position at Princeton to serve under Veblen at the Aberdeen Proving Ground. That same year, he married Natalia Levitzkaya, a White Russian whom he met in Paris.

At the end of World War I, Alexander returned to Princeton, where he focused exclusively on topology for the rest of his career. He spent the next ten years publishing papers on the classification of manifolds. In 1920, he proved the Jordan curve theorem, which led to the Alexander duality theorem in 1922. This theory in turn led to developments in homology theories with different coefficients and in cohomology theory. Another creation in this period was the Alexander horned sphere. In the late 1920s, he started working on knot theory, and discovered in 1928 that for a matrix of polynomials associated with a knot the equivalence class is an invariant of the knot type. Using this, he extracted a series of polynomials, including one named for him, which is such a sensitive invariant that it easily distinguishes most knots found in knot tables compiled in the nineteenth century.

In 1924, Alexander persuaded Veblen to bring Solomon Lefschetz, another prominent topologist, to Princeton from Kansas. Lefschetz, Veblen, and Alexander together were a powerful group, and were able to bring many students and visitors from Europe to Princeton, including several leading European researchers. In 1933 Veblen and Alexander were recruited at the Institute for Advanced Study, along with Einstein, von Neumann, and Weyl. Alexander's last major finding was a formula for a product in cohomology, which was announced in a conference in Moscow in 1935 at the same time that Kolmogorov announced a very similar formula in the same field.

In World War II, Alexander worked as a civilian with the Office of Scientific Research and Development for the Army Air Force. After the war, he became more and more reclusive. In 1947, he published his last paper, and the following year he asked to become a non-stipendiary permanent member of the Institute instead of a professor. In 1951, he retired from the Institute and practically disappeared from public life. He did not even attend a symposium in honor of Lefschetz' retirement in 1953. After his wife's death in 1967, his health started to decline. He died of pneumonia on September 23, 1971, at age 83.

While Alexander was one of the leading topologists of his time, he never had a sense of competitiveness, possibly due to his affluent upbringing. Work was more a matter of enjoyment than a way to make a living. He was an avid mountain climber, and has over 200 climbs on record in the Alps. He enjoyed climbing so much that he often scaled buildings on Princeton's campus, and it was general knowledge that he preferred entering his top-floor office through the window rather than the door. Alexander was well liked by both faculty and students, and was widely regarded as one of the best lecturers at Princeton.

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