

American Mathematical Advancements in World War I

On January 4, 1918, Dr. Oswald Veblen, a mathematician at Princeton University, took command of the office of experimental ballistics at Aberdeen, Maryland. The proving ground at Aberdeen was the final stop that most weapons made on the way to the Western Front. The office of experimental ballistics studied the flight of shells and the force of explosions and generated range tables. These tables, which presented the range of a gun as a function of muzzle velocity, barrel angle, and type of shell, were not used by soldiers, since checking tables in the heat of battle was impractical, but by engineers and weapons designers. Soon after he arrived at Aberdeen, Veblen started to assemble a team of mathematicians from Princeton, Harvard, Columbia, and the University of Chicago. Even though the work at Aberdeen was hardly as attractive as work at a university, and military life imposed restrictions on the academic freedom that most professors enjoyed, mathematicians signed up in droves. Sixty served at some point under Veblen at Aberdeen, and ninety others fought, worked as surveyors or cartographers, or taught trigonometry to officer candidates.

Veblen started by working on range tables for a 2.95-inch lightweight mountain gun that could be carried on horseback. Unfortunately, since the ranges at Aberdeen were not ready when he started his research, he had to fire the guns into an empty field and use relatively primitive tools to measure range, then ride down to the range on horseback during meal breaks and mark the shell holes so they could be identified by surveyors. The winter weather, which was particularly harsh, did not help much (although the constant snowfall at least filled old craters with fresh deposits of snow). All told, it took over four weeks for the crews to gather enough data to proceed.

Veblen used a differential equation model developed by the Italian mathematician Francesco Siacci, which expresses the forces influencing a projectile as a nonlinear function of altitude and velocity, to construct the range tables. The model was written as

$$\begin{aligned}\delta^2x/\delta t^2 &= -E(y, v) \delta x/\delta t \\ \delta^2y/\delta t^2 &= -E(y, v) \delta y/\delta t - g\end{aligned}$$

where t is time, x and y are horizontal and vertical position, respectively, v is velocity, and g is acceleration due to gravity. The initial conditions are

$$\begin{aligned}x_0 &= y_0 = 0 \\ \delta x_0/\delta t &= v_0 \cos(\theta_0) \\ \delta y_0/\delta t &= v_0 \sin(\theta_0)\end{aligned}$$

where θ_0 is the angle of the gun barrel and v_0 is the velocity of the projectile at the muzzle of the gun. $E(y, v)$ is given by

$$E(y, v) = H(y)G(v) / C$$

where $H(y)$ gives the ratio of the air density at altitude y to the standard density at sea level. In 1918, the ordnance staff used $H(y) = e^{-.0001036y}$. $G(v)$ gives the drag on a projectile as a function of velocity, and was given by a piecewise polynomial. Other factors such as shape, size, and mass of the shell were included in C , called the *ballistics coefficient*, which was used as a fudge factor and was determined empirically. Since $G(v)$ was a piecewise function, which made solving the differential equations in closed form impossible, Veblen used four ballistics functions, also developed by Siacci, that would enable him to approximate the solution. These

functions were called space, time, inclination, and altitude functions, denoted S , T , I , and A , respectively. The functions involved were:

$$t = C \sec(\theta_0)[T(u) - T(v_0)] \quad (1)$$

$$x = C[S(u) - S(v_0)] \quad (2)$$

$$y/x = \tan(\theta_0) - (C/2) \sec^2(\theta_0) [(A(u) - A(v_0))/(S(u) - S(v_0)) - I(v_0)] \quad (3)$$

$$\tan(\theta) = \tan(\theta_0) - (C/2) \sec^2(\theta_0) [I(u) - I(v_0)] \quad (4)$$

where u is called the pseudo-velocity and is given by

$$u = v \cos(\theta) \sec(\theta_0)$$

and θ is the angle at which the shell is traveling.

Veblen would estimate C by setting the barrel of the gun parallel to the ground, so that $\theta_0 = 0$, $\tan(\theta_0) = 0$, and $\sec^2(\theta_0) = 1$. The muzzle velocity and range could be easily measured, from which the pseudo-velocity at the point of impact (u_T) could be calculated from the equation

$$I(v_0) = (A(u_T) - A(v_0)) / (S(u_T) - S(v_0))$$

which comes from (3). Once u_T had been computed, C could be calculated using (2).

Estimating C took much longer than expected. Veblen planned on having three mathematics instructors, lieutenants stationed at Fort Hancock, 150 miles north of Aberdeen, compute C , but soon learned that long-range correspondence was not as easy as he had expected. The three lieutenants found numerous problems in the data, sent numerous telegrams to Aberdeen, and eventually requested that Veblen come to Fort Hancock to help them. Finally, on March 25, 1918, they solved all problems with the data, and then worked through the night to complete the tables.

After working on range tables for the mountain gun, Veblen assisted the newly-formed office of mathematical ballistics in Washington DC with ballistics research, helped them hire mathematicians, worked on collecting data to test new models set up to take wind and the earth's rotation into account when generating range tables, and hired human computers to work on range tables, ballistics reports, and tables of special functions. While the ballistics staff had started to disperse in September, the remaining staff was still starting to work on new problems when orders came on November 11, the end of the war, to stop work. Veblen, however, had just been sent to Europe to inspect European ballistics labs. Gradually, he started shifting his interest from ballistics back to mathematics. After he was discharged, he served as president of the American Mathematical Society. Later he was one of the first faculty members at the Institute for Advanced Study and served on the Applied Mathematics Panel of the Office of Scientific Research and Development during World War II. While the other mathematicians were not as famous, and few continued to work in ballistics, they remained a community. This was probably the biggest effect that Aberdeen had—not its impact on World War I or any research into artillery that might have been done there, but the fact that it created a community out of a generation of mathematicians.

Source: David Alan Grier, Dr. Veblen takes a uniform: Mathematics in the First World War, *American Mathematical Monthly* **108** (2001) 922-931.

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April 26, 2004