↑name↑ ↑ID#↑ February 20, 2003. (2 pages.)

Math 55 – Spr 2002 – Quiz 1 ANSWERS – Warren D. Smith

1. **Circle all correct answers among a,b,c,d,e,f. There is always at least 1, but may be MORE than 1, correct answer per problem, in which case circle ALL correct answers!)** Which of the following is an *exponential* growth (or decay) relationship?
(a) Temperature increasing 3°C per hour. (b) Number of bacteria increasing 30% per hour. (c) Number of cars in park decreasing 5 cars per hour. (d) Price of houses increasing \$1500 per year. (e) Each year, number of racist politicians shrinks to only 95% of their number the preceding year.
ANS. b and e. *Exponential* growth means you repeatedly *multiply* a quantity by some constant (here 1.3 for the bacteria, but 0.95 for the politicians). *Linear* growth is you repeatedly *add* a constant (such as 3 for the temperature & -5 for the cars).

2. Enrollment at a school is 2700. If enrollment increases each year by 9%, how many students will be there 19 years later?
ANS. a: about $2700 \times 1.09^{19} = 13882.49$. Key word is *each*. First year: $2700 \to 2700 \times 1.09 = 2943$. Second year: $2943 \to 2943 \times 1.09 \approx 3208$. Etc.

3. The price of rubber is \$5.70 now, but 10 years ago was \$3.20. Assuming a linear relationship between price and number of years, what do you estimate the price will be in 7 years?
ANS. c: $7 \times (5.70 - 3.20)/10 + 5.70 = 7.45$. Use formula G. Slope of line is $(5.70 - 3.20)/10 =$ rise/run $=$ 0.25. y-intercept is 5.70. So the line is $y = Sx + K = 0.25x + 5.70$, where y is price of rubber and x is time in years after now. If we put $x = 7$ to go 7 years into the future, we predict $y = 0.25 \times 7 + 5.70 = 7.45$.

4. You put \$1000 in an account with APR 5.3%, compounded quarterly. How much is there in 3 years?
ANS. c. $1000 \times (1 + .053/4)^{12} = 1171.11$. Use formula C.

5. You contribute \$200 per month to an IRA earning 6% APR compounded monthly. After 20 years, how much money will be there?
ANS. d: $200 \times \frac{(1+0.06/12)^{12*20}-1}{0.06/12} = 92408.18$. Use formula A.

6. If $x \geq 0$ and $R = 9x^2 + 5$, then what is x? ANS c: $x = \frac{\sqrt{R-5}}{3}$
Subtract 5 from both sides. Take square root of both sides. Then divide by 3. Also x equal to minus this would have been an answer, except I said "$x \geq 0$."

7. A population doubles every 20 years. By what factor does it grow in 100 years? ANS d: $32 = 2^5$ (100 years is 5 times 20 years so there are 5 doublings.)

8. You want your daughter to have \$135000 in 13 years to pay for college. You can get an APR of 8.7% compounded monthly. How much should you deposit at the end of each month?
ANS a: $135000 \times (0.087/12)/((1 + 0.087/12)^{12\times 13} - 1) = 469.17$. Use formula B.

9. You invest \$3000 in a mutual fund. Over 4 years its value grows to \$8400. What was the equivalent annual percentage yield? ANS c: $(8400/3000)^{(1/4)} - 1 = 0.293568 = 29.4\%$. Use formula E. The final yield after all 4 years was 8400/3000=280%, minus the 100% you started with, for (as the yield) 180% increase (or as a *factor* 280%). However, question did not ask for that – it asked for the *annualized* yield.

10. You borrow \$10000, and you pay it off over 10 years at 10% APR, one fixed-size payment per year. How much do you pay per year?
ANS c: Use loan payment formula F: PMT$= 10000 \times 0.1/(1 - (1 + .1)^{-10}) = 1627.45$.

11. The number of cars that park in Joe's lot is 200 on the zeroth day, but then each day 3 more cars park there than on the day before. How many cars are parked there on day n? ANS c: $200 + 3n$

12. During the first 50 days of operation of Joe's car parking lot (previous question), combined, the total number of cars that park there is? ANS e: Use formula I with $S = 20$ and $F = 200 + 49 \times 3 = 347$ and $n = 50$ so the total cars on days 0,1,2,...,49 are $\frac{50}{2}(200 + 347) = 13675$.

13. You get a fixed rate mortgage for a \$50000 house. Closing costs are \$1000. The mortgage comes with 3 "points." Zero down payment. Then you have to pay it off at a fixed amount per month, for 10 years. Interest is 2% APR compounded monthly. What are your total payments during those 10 years, including all principal, interest, points, and costs?
ANS: a: your payment per month is $50000 \times (.02/12)/(1-(1+.02/12)^{-10\times12}) = \$460.07$ by formula F. This times 10×12 is 55208.06 (total payments over all 10×12 months) This plus 1000 (for costs) plus $.03 \times 50000 = 1500$ (for 3 points) is ANS=\$57708.06.

14. What is $3^{99} \times 3^{999}$? ANS e: $3^{99+999} = 3^{1098}$. Because $A^B A^C = A^{B+C}$.

15. The half-life of carbon-14 is 5730 years. Tests show an Egyptian mummy has only 1/16 of the carbon-14 percentage that a recently dead Egyptian has. How many years ago did the mummy die? (Actually everything has been screwed up by 1950s nuke test fallout. Ignore that.)
ANS d: $5730 \times 4 = 22920$ because $2^4 = 16$ so need 4 halvings, which takes 4×5730 years.

16. $5x^7 = 9$. What is x? ANS e: $x = (9/5)^{1/7}$ (divide by 5 on both sides then take 1/7 power using $(A^B)^C = A^{BC}$ with $B = 7$ and $C = 1/7$ so $BC = 1$, and noting $x^1 = x$.

17. In that mummy's tomb, you find records showing your family owes the mummy's family for an unpaid loan of \$1000, that has been accumulating interest of 1% APR (compounded annually) for the last 6666 years. You know that means trouble. How much is the debt now (accurate to 1%)?
(a) $\$6.4 \times 10^{32}$; (b) \$7666; (c) \$1666.60; (d) More than all money in world; (e) $\$6.4 \times 10^{31}$.
ANS By formula C, total debt is $1000 \times (1.01)^{6666} = 6.4 \times 10^{31}$. Both d and e correct, because the world population is 6×10^9 (6 billion) and even if everybody were a millionaire ($\$10^6$) the total money in the world would be at most $6 \times 10^9 \times 10^6 = 6 \times 10^{9+6} = 6 \times 10^{15}$ which is way way smaller than 10^{31}.

18. Economists sometimes have crazy politicized ideas. During his Nobel prize acceptance speech, one remarked, in the course of praising the glories of capitalism, that "real [that is, adjusted for inflation, i.e. everything re-expressed in today's dollars] per capita wages have been increasing at 5% per year ever since the time of the Ancient Greeks, and I see no reason this cannot continue forever." Let's check his math. Assume the per capita annual wage now is \$20000. What real annual wage was a typical Ancient Greek earning 2500 years ago?
(a) $\$20000 \times (1.05)^{-2500}$; (b) $\$20000/(1.05)^{2500}$; (c) Less than 1 penny per year; (d) Less than 1 atom of the metal in 1 penny per year; (e) About \$1; (f) About \$10.
ANS a,b,c,d all correct. You can see a and b are the same thing since $A^{-B} = 1/A^B$. Computing either, we get 2.1×10^{-49} dollars per year as the Ancient Greek "salary" which is way smaller than 1 penny. In fact the number of metal atoms in a penny is around 10^{22} (chemistry: Avogadro's number is 6×10^{23} is 1 "mole;" another way to see this is to know the size of an atom is about 10^{-10} meters; very crude estimates are good enough for our purposes) so this is way less than even 1 atom of 1 penny.

    Obviously this Nobel Prize winning economist was worse at math than you are. Capitalism may be wonderful, but it is *not* wonderful enough to generate exponential growth forever. Nothing is. Capitalism is not even wonderful enough to sustain even 1% real wage growth over the last 2500 years. (0.1% may be valid.) We are now in an exponential growth era, but as we've seen in the homework, that growth can't keep going for very long.

19. **Here's a list of hopefully useful formulas.**

    A. Savings plan formula: A =amount at end, PMT=periodic deposit, i =interest rate (APR), n =number of deposits (and interest compoundings) per year, Y =number of years: $A = PMT \times \frac{(1+i/n)^{nY}-1}{i/n}$.

    B. Same thing but solved for PMT: $PMT = A \times \frac{i/n}{(1+i/n)^{nY}-1}$.

C. Compound interest: $A = P \times (1+i/n)^{nY}$ where P =principal, A =amount at end of Y years, interest rate i (APR), compounded n times per year.

D. Simple interest: $A = P \times (1+iY)$ where P =principal, A =amount at end of Y years, interest rate= i (APR).

E. Annualized yield: If some investment of P ends up with A dollars Y years later, then $A = P \times (1+i)^Y$ where i is the equivalent annual percentage yield (as a decimal). Solved for i this is $i = (A/P)^{1/Y} - 1$.

F. Loan payment formula: PMT= $P\frac{i/n}{1-(1+i/n)^{-nY}}$.

G. y is a linear function of x: $y = Sx + K$ where S is slope=rise/run and K is the y-intercept. If have the (x, y) coordinates of two points (a, b) and (c, d): the line thru them has slope $S = (d-b)/(c-a)$ =rise/run and the y-intercept K is chosen so that the line goes thru the points, e.g. so that $Sa + K = b$.

H. y is an exponential function of n: $y = Sc^n$ where S is the value of y when $n = 0$ (y-intercept) and c is the "growth rate" ($c > 1$ for growth, $c < 1$ for decay).

I. Sum of n linearly growing (or decaying) quantities: The quick way to add up S, $S+I$, $S+2I$, $S+3I$, etc. (n terms in all) is sum= $(S + F)\frac{n}{2}$ where S is the start and $F = S + (n-1)I$ is the final term.

J. Sum of n exponentially growing (or decaying) quantities: The quick way to add up S, Sc, Sc^2, Sc^3, etc. (n terms in all) is sum= $S\frac{1-c^n}{1-c}$ where S is the start and c is the growth (or decay) ratio.

NOTE. If you circle a wrong answer, you get zero. If you circle c correct answers, but there are B correct answers with $c \leq B$, then you get c/B.