↑name↑ ↑ID#↑ April 17, 2003. (2 pages.)

Math 55 – Spr 2002 – PRACTICE QUESTIONS FOR Quiz 2.2 – W.D. Smith.

1. **These are a few examples of especially hard (?) probability questions which might be similar to the hardest ones on the upcoming (Tues 22 Apr) quiz 2 (second try). You also have the answers to quiz 2 (first try). If you get comfortable with all of those, plus these, you'll do well on the quiz!**

2. You pick 2 cards from a standard 52-card, 4-suit deck. What are the *odds* your two cards happen to add up to 11 (Ace counts as 1, Jack as 11, Queen as 12, King as 13)?

   ANS. There are 10 ways the first card plus the second card could add up to 11, namely $1 + 10$, $2 + 9$, $3 + 8$, $4 + 7$, $5 + 6$, $6 + 5$, $7 + 4$, $8 + 3$, $9 + 2$, $10 + 1$. *And* each card could be any of 4 suits, so that is $10 \times 4 \times 4 = 160$ possible 2-card lineups that add up to 11. Meanwhile the number of ways to pick a first card then a second card from the deck is ${}_{52}P_2 = 52 \times 51 = 2652$. (Note: I have computed both numbers 160 and 2652 under the assumption that the two different orders XY and YX of the two cards X and Y in your hand, count as different. You have to use the *same*, *compatible* assumption, is key. If you had said you won't count different orders are different, then the two counts would have been ${}_{52}C_2 = 52 \times 51/2 = 2652/2 = 1326$ and $10 \times 4 \times 4/2 = 160/2 = 80$ and the ratio would come out the same and it would still work.) So the ratio of our two counts, which is the probability when you pick the 2 cards, they will add to 11, is $p = 160/2652 = 50/663 \approx .07541478$. *But* the problem asked for the *odds* (not the *probability*) so we have to compute $p/(1 - p) = (50/663)/(613/663) = 50/613 \approx .081566$ as the ANSWER. [Also this is $.07541478/(1 - .07541478) = .081566$.]

3. You ask 3 people to give you a dollar. The probability the first one says yes is .05, the probability the second says yes is .03 and the probability the third says yes is .02. (All independent since you don't let any know you asked the others.) **What** is the probability that you get at least a dollar?
   ANS. This is the same as the prob. that it does NOT happen that {person 1 AND person 2 AND person 3 all say no}. So using the NOT rule and the AND-multiply rule for independent events, this prob is $1 - (.95 \times .97 \times .98) = .096930 = 9.69\%$.

4. (continued) What is the expected amount of money you got from them?
   ANS. Expected dollars from first person was $.05 \times 1 + .95 \times 0 = .05$. Expected dollars from second person was $.03 \times 1 + .97 \times 0 = .03$ Expected dollars from third person was $.02 \times 1 + .98 \times 0 = .02$. Since expected values add, the total expected value was $.05 + .03 + .02 = .10$, i.e. 10 cents. Note this is a little more than 9.69 cents because there is a small chance you'll get more than 1 dollar.

5. You roll a dice and a coin. What is the probability the results add up to 4 (dice results: 1-6; coin: heads=0 or tails=1)?
   ANS. #ways they can add up to four is 2, since ways are $0 + 4$ (heads&4) and $1 + 3$ (tails&3). #ways they can come out is $2 \times 6 = 12$. Ratio = probability = $2/12 = 1/6$ = ANSWER.

6. (continued) What is the expected sum of the dice and the coin results?
   ANS. Expected coin result $= \frac{1}{2} \times 1 + \frac{1}{2} \times 0 = \frac{1}{2}$. Expected dice result $= \frac{1}{6} \times (1+2+3+4+5+6) = 21/6 = 3.5$. Since expectations add, expected total result is $3.5 + .5 = 4$.

7. (continued) What is the probability {the dice comes up 5 OR 6} AND the coin heads?
   ANS. The dice comes out 5 or 6 with probability 2/6=1/3. (Or use the OR-add rule for disjoint events to get $1/6 + 1/6 = 1/3$.) The coin is heads with probability 1/2. The probability of both (AND-multiply rule, independent events) is $(1/2) \times (1/3) = 1/6$.

8. If you bet \$1 that an event of probability 1/5 will occur, then what should your winnings be (if you win) to make it a fair bet?
ANS. Odds are (1/5):(4/5), i.e. 1:4, so you should win \$4 or lose \$1 to be fair.

9. If you pick a card from a deck, AND you roll a dice, what is the probability the dice comes up 5 AND the card is an Ace} OR {the dice comes up 1}?
ANS. 5 AND ace: prob=$(1/6) \times (1/13) = 1/78$. (AND-multiply rule useable since independent events since card can't affect dice.) dice=1: prob=(1/6). Now, are these two events *disjoint*? Yes, because they cannot both happen (if dice=1, then dice cannot be 5). So can use disjoint OR-add rule. ANSWER $(1/78) + (1/6) = 7/39 \approx .179487$.

10. You pick a card from a deck, AND you roll a dice, what is the probability {the dice comes up 6 AND the card is a King} OR {the dice comes up 6}?
ANS. This OR is *not* for disjoint events. So can't use the probability-add-rule. But really, this problem is simple, because really it is just asking "what is the probability the dice comes up 6?". The card is actually irrelevant to whether the total event happens. So the answer is 1/6.

11. If the probability it will rain on a random day is 5%, then what is the probability that, on a random day AND on the day after that, it will rain both times?
ANS. No way to tell!! These two events are *dependent*, and we have not been told how they depend on each other. So the answer is NOT $.05 \times .05 = .0025$.

12. (continued) what is the probability that it will rain on some random day you pick (by throwing a dart at next year's calendar) to have a party, OR, when you throw a dice, you will get 6?
ANS. These events not disjoint (it could both rain and you could roll a 6) so cannot use OR-add rule. So we use the Pr(A OR B) = Pr(A) + Pr(B) - Pr(A AND B) rule (subtracting off the overlap probability). Now since dice and rain are independent, can use AND-multiply rule. So ANSWER is $.05 + (1/6) - (1/6) \times .05 = .208333333$.

13. You bet on 3 independent horse races, with odds 1:6, 2:3, and 5:6. Assume unrealistically that those all were fair odds! What is the probability you win all three bets?
ANS. The *probabilities* (as opposed to *odds*) of winning the three races are 1/7, 2/5, and 5/11. Now since the probabilities are independent we can use the AND-multiply rule to get ANSWER=$(1/7) \times (2/5) \times (5/11) = 10/385 = 2/77 \approx .025974$.

14. In a certain lottery, you guess a 3-digit number such as 090 or 673. You pay \$1 per guess i.e. per ticket. (0's are allowed as digits.) If you get the number completely right, you get \$500. Otherwise, if you got 2 consecutive digits right, then you get \$50. What is the expected lottery win from one of these tickets? Is it wise to buy such a ticket?
ANS. Prob(get all 3 right)= $1/1000 = (1/10) \times (1/10) \times (1/10) = .001$.
Prob(get first 2 right but last wrong)= $(1/10) \times (1/10) \times (9/10) = 9/1000 = .009$.
Prob(get last 2 right but first wrong)= $(9/10) \times (1/10) \times (1/10) = 9/1000 = .009$.
Expected winnings are then $.001 \times 500 + .009 \times 50 + .009 \times 50 + .981 \times 0 = 1.40$. WOW!! Here you actually expect to win more than the \$1 cost of the ticket – buying a ticket is like getting a free gift of 40 cents!! So yes, buying such a ticket (in fact, as many as you can) *would* be wise. (Warning: good luck trying to find such a lottery in the real world...)