↑name↑ ↑ID#↑ May 7, 2003.

Math 127(3) – Spr 2003 – PRACTICE Final – Warren D. Smith.

1-page "crib sheet" allowed. Calculators allowed. Books and notes NOT allowed. Show all work.

1. Consider $F(x,y) = \sqrt{x^2 - 3y}$. What is the *domain*-set of this function? Sketch that set.
2. And what is its *range* set?
3. What is the level set $F = 4$. Sketch it.
4. What is the gradient $\vec{\nabla} F$ of F?
5. What is the directional derivative of F at $(x,y) = (2,1)$ in the direction $(1,1)/\sqrt{2}$?
6. What is the angle between the gradient-vector of F at the point $(x,y) = (-5,3)$, and the level set $F = 4$?
7. A parallelogram in 3-dimensional space has 3 vertices $(0,4,7)$, $(1,1,2)$ and $(1,0,-1)$. What is its 4th vertex?
8. (Same parallelogram) What is the angle at that 4th vertex?
9. (Same parallelogram) What is its area?
10. For $H(x,y) = x^3 + 2xy - 2y^2$, find all local minima, local maxima, and saddle points (and say which are which).
11. Maximize $F(x,y,z) = 2x + 3y - 5z$ subject to the constraint $z = x^2 + y^2$. [Hint: Lagrange multiplier trick may help.]
12. A parallelipiped has these edge-vectors emanating from one of its vertices: $(3,-1,4)$, $(-1,2,5)$, and $(1,7,-1)$. What is its volume?
13. For each of the following decide if it is a vector, a scalar, or senseless:
    $(\vec{a} \times \vec{b}) \times \vec{c}$.
    $(\vec{a} \cdot \vec{b}) \times \vec{c}$.
    $(\vec{a} \times \vec{b}) \cdot \vec{c}$.
    $(\vec{a} - \vec{b}) \cdot \vec{c}$.
    $(\vec{a} - \vec{b}) \times \vec{c}$.
    $(\vec{a} \cdot \vec{b}) - \vec{c}$.
    $(\vec{a} \cdot \vec{b}) + 3$.
    $(\vec{a} \cdot \vec{b}) + \vec{c}$.
    $(\vec{a} \times \vec{b}) + 3\vec{c}$.
14. Here is a curve: $\vec{r}(t) = (3t,\ 9 + t^2)$. Find all points $(x,y) = \vec{r}(t)$ on this curve such that $\vec{r}(t)$ and $\vec{r}'(t)$ are (i) perendicular (ii) parallel in same direction (iii) parallel in opposite directions.
15. Parameterize the curve $5x^2 + 7y^2 = 54$ with a single, everywhere-smooth function of t, going clockwise as t increases.
16. Write an integral giving the length of that curve (but do not evaluate the integral).
17. Simplify $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{a} + \vec{c}) + \vec{c} \times \vec{b}$?
18. What is the curl of $(x^2, x + y + z, 5)$?

19. What is the div of $(x^2, x+y+z, 5)$?

20. Here is a double integral. $\int_0^9 \int_{\sqrt{y}}^3 \cos(x^3) \mathrm{d}x\mathrm{d}y$. Evaluate it completely. [Hint. What happens if you interchange the order of integration? What are the right bounds on the integrals in that case? May help to sketch the region of integration.]

21. What function, if any, has gradient $(5x,\ 2y,\ 7+\cos z)$?

22. What is the curve integral $\int (5x, 2y, \cos(z)+7) \cdot \mathrm{d}\vec{s}$ along *any* curve from $(-1, 4, 5)$ to $(2, 0, 0)$?

23. What is $\int\int\int (7+x+xy+x^2) \mathrm{d}x\mathrm{d}y\mathrm{d}z$ over the $2 \times 4 \times 6$ brick $|x|<1$, $|y|<2$, $|z|<3$?

24. What is the flux of the smooth vector field $\vec{F}(x, y, z) = (x, y, z + \arctan(5+xy))$ out through the surface of that brick?

25. Let $\vec{H}(x, y) = (xy+3+\cos x,\ xy+\sinh y)$. What is the circulation integral $\int \vec{H} \cdot \mathrm{d}\vec{s}$ anticlockwise around the closed curve bounding the region $3x^2 < y < 12$ in the xy plane. [Hint: Green or Stokes.]

26. What is the vector element of surface area of the parameterized surface $(x, y, z) = (\sin p,\ \cos q,\ p+q)$?

27. What is the surface area (over the parameter region $0 < p < \pi/2$, $0 < q < \pi/2$)?