↑name↑ ↑ID#↑ May 13, 2003. (2 pages.)

Math 127(3) – Spr 2002 – PRACTICE Quiz 2 – Warren D. Smith.

1. **This test may cover the following...** length of a curve; Parametrizations of curve-segments; Directional derivative in 3D; gradient; and the fact that gradients, as directions of steepest increase, are perpendicular to (normal to) level surfaces; curl; divergence; line integrals, energy and work; double integral, change in integration order; Jacobian; spherical & cylindrical coords; parameterizations of surfaces; Area of a surface; Flux and the divergence theorem; circulation and Stokes theorem. Probably it will mostly be on the lattermost portion of this list so we can test newer stuff more. **1-page "crib sheet" allowed. Calculators allowed.** Books and noted NOT allowed. **Example questions follow.**

2. Find the length of the curve $(x, y, z) = (4t, 3t + 1, 7)$ from $t = 0$ to $t = 1$.
ANS. $\int_0^1 \sqrt{4^2 + 3^2} dt = 5$.

3. Find a parameterization of the curve $y^2 + x^3 = 1$.
ANS. Simplest is to use y as the parameter. $x = (1 - y^2)^{1/3}$, $y = y$, for $-\infty \le y \le \infty$.
Note, using $y = \sqrt{1 - x^3}$ would be less good since you'd really have to paste together two curve segments, one the $+\sqrt{}$, the other for the $-\sqrt{}$ to get the whole curve.

4. Let $\vec{F}(x, y, z) = (yz, \sin(xz), e^{xy})$. What is its curl $\vec{\nabla} \times \vec{F}$?
ANS $(e^{xy}x - \cos(xz)x,\ y - e^{xy}y,\ \cos(xz)z - z)$.

5. And what is its divergence $\vec{\nabla} \cdot \vec{F}$?
ANS. $0 + 0 + 0 = 0$.

6. What is the line integral $\int \vec{F} \cdot d\vec{s}$ along the line-segment from $(0, 0, 0)$ to $(1, 1, 1)$? Express as an ordinary integral but don't do it.
ANS. $\int_0^1 (t^2, \sin(t^2), \exp(t^2)) \cdot (1, 1, 1) dt = \int_0^1 [t^2 + \sin(t^2) + \exp(t^2)] dt$

7. Consider the surface $x^3 + y^2 + z = 1$. What is a unit normal to this surface at a point (x, y, z) on it?
ANS. $(3x^2, 2y, 1)/\sqrt{9x^4 + 4y^2 + 1}$.

8. What would be a parameterization of that surface?
ANS. Use x and y as parameters. $x = x$, $y = y$, $z = 1 - x^3 - y^2$, for all real x,y.

9. What would be the surface area of that surface where it intersects the infinite triangular prism $0 < x < 1$ and $0 < y < 1$ and $x + y < 1$? Write fully explicitly as a double integral but do not actually do either the inner or outer integral.
ANS. $\int_{x=0}^{x=1} \int_{y=0}^{y=1-x} \sqrt{1 + (3x^2)^2 + (2y)^2}\, dy dx$.

10. Compute the Jacobian matrix of this change of variables:
$X(a, b, c) = a + b$, $Y(a, b, c) = a - b$, $Z(a, b, c) = c$.
ANS.

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1  1 0
1 -1 0
0  0 1
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11. And its determinant is?
ANS. -2.

12. So $dx dy dz = K(a, b, c) da db dc$, for what volume-adjustment function $K(a, b, c)$?
ANS. $K(a, b, c) = -2$.

13. Evaluate $\int\int\int 5\,dxdydz$ over the region $1 \le x^2+y^2+z^2 < 4$, $x > 0$.
ANS. Realize region is a hemisphere of radius 2 with a hemisphere hole of radius 1. Ans is $5 \times$ volume. And volume is $\frac{1}{2}\frac{4\pi}{3}(2^3-1^3)$. So Ans is $\frac{70\pi}{3}$.

14. Suppose $\vec{F}(x,y,z)$ is a function whose curl is $\vec{\nabla} \times \vec{F} = (x,y,z)$. What is the curve-integral of $\int \vec{F} \cdot d\vec{s}$ around the unit circle curve $x^2+y^2=1$, $z=0$ going anticlockwise?
ANS. Use Stokes theorem to see it is the same as the surface integral of $\int\int (x,y,z) \cdot d\vec{area}$ on the unit hemisphere $x^2+y^2+z^2=1$, $z>0$. Since the outward unit normal vector $\vec{u}$ to the sphere is also (x,y,z), and since by definition $d\vec{area} = \vec{u}darea$, and since $(x,y,z)\cdot(x,y,z)=1$, this integral is $\int\int 1darea =$ area $= \frac{1}{2}4\pi r^2 = 2\pi$.

15. What is the flux of (x,y,z) out of the radius-1 height-3 cylinder (curved part only, ignore the flux out of flat ends of the cylinder) $x^2+y^2 \le 1$, $0 < z < 3$?
ANS. Unit outward normal is $(x,y,0)$. Flux is this Integral: $\int\int (x,y,z)\cdot(x,y,0)darea = \int\int (x^2+y^2)darea = \int\int 1darea = \text{area} = 2\pi \cdot 3 = 6\pi$.

16. In the previous problem, find the flux out of the whole cylinder (this time *including* both its flat circular endcaps).
ANS. Using the divergence theorem and the fact $\vec{\nabla} \cdot (x,y,z) = 1+1+1 = 3$, we get that the flux is $\int\int\int 3dxdydz = 3\text{volume} = 3 \cdot 3\pi = 9\pi$.

SECOND ANS. Without using divergence thm, we note the outward unit normal vector to the top endcap $(z=3)$ is $(0,0,1)$, so the flux out thru that endcap is $\int\int (x,y,z)\cdot(0,0,1)darea = \int\int z darea = 3\text{area} = 3\pi$.

The outward unit normal vector to the bottom endcap $(z=0)$ is $(0,0,-1)$, so the flux out thru that endcap is $\int\int (x,y,z)\cdot(0,0,-1)darea = \int\int z darea = 0\text{area} = 0$.

So the total flux is $6\pi + 3\pi + 0 = 9\pi$.