↑name↑ ↑ID#↑ April 11, 2003. (2 pages.)

Math 127(3) – Spr 2002 – PRACTICE Quiz 2 – Warren D. Smith.

1. **This test may cover the following...** length of a curve; Parametrizations of curve-segments; Directional derivative in 3D; gradient; and the fact that gradients, as directions of steepest increase, are perpendicular to (normal to) level surfaces; curl; divergence; line integrals, energy and work; double integral, change in integration order; Jacobian; spherical & cylindrical coords; parameterizations of surfaces; Area of a surface; Flux and the divergence theorem; circulation and Stokes theorem. Probably it will mostly be on the lattermost portion of this list so we can test newer stuff more. **Example questions follow.**

2. Find the length of the curve $(x, y, z) = (4t, 3t + 1, 7)$ from $t = 0$ to $t = 1$.

3. Find a parameterization of the curve $y^2 + x^3 = 1$.

4. Let $\vec{F}(x, y, z) = (yz, \sin(xz), e^{xy})$. What is its curl $\vec{\nabla} \times \vec{F}$?

5. And what is its divergence $\vec{\nabla} \cdot \vec{F}$?

6. What is the line integral $\int \vec{F} \cdot \mathrm{d}\vec{s}$ along the line-segment from $(0, 0, 0)$ to $(1, 1, 1)$? Express as an ordinary integral but don't do it.

7. Consider the surface $x^3 + y^2 + z = 1$. What is a unit normal to this surface at a point (x, y, z) on it?

8. What would be a parameterization of that surface?

9. What would be the surface area of that surface where it intersects the infinite triangular prism $0 < x < 1$ and $0 < y < 1$ and $x + y < 1$? Write fully explicitly as a double integral but do not actually do either the inner or outer integral.

10. Compute the Jacobian matrix of this change of variables:
$X(a, b, c) = a + b$, $Y(a, b, c) = a - b$, $Z(a, b, c) = c$.

11. And its determinant is?

12. So $\mathrm{d}x\mathrm{d}y\mathrm{d}z = K(a, b, c)\mathrm{d}a\mathrm{d}b\mathrm{d}c$, for what volume-adjustment function $K(a, b, c)$?

13. Evaluate $\int\int\int 5\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$ over the region $1 \le x^2 + y^2 + z^2 < 4$, $x > 0$.

14. Suppose $\vec{F}(x, y, z)$ is a function whose curl is $\vec{\nabla} \times \vec{F} = (x, y, z)$. What is the curve-integral of $\int \vec{F} \cdot \mathrm{d}\vec{s}$ around the unit circle curve $x^2 + y^2 = 1$, $z = 0$ going anticlockwise?

15. What is the flux of (x, y, z) out of the radius-1 height-3 cylinder (curved part only, ignore the flux out of flat ends of the cylinder) $x^2 + y^2 \leq 1$, $0 < z < 3$?

16. In the previous problem, find the flux out of the whole cylinder (this time including both its flat circular endcaps).