↑name↑ ↑ID#↑ December 5, 2002. Show all work.

Math 75 – Fall 2002 – Hmwk #9 (psuedo-final) – Warren D. Smith

Due: first class after thanksgiving. This homework is quite similar to what the real final exam (2 hours long) will be like. Feel free to use extra sheets, the real final would have a lot more white space.

1. Find derivative of these: $\frac{x^3}{4} - \frac{3}{x} + 5\sqrt{x} - \sin(x^2)$

   ANS: $\frac{3x^2}{4} + \frac{3}{x^2} + \frac{5}{2}x^{-1/2} - 2x\cos(x^2)$

2. $\frac{x^2+1}{x^3+7}$

   ANS: $\frac{(2x)(x^3+7)-(x^2+1)(3x^2)}{(x^3+7)^2}$

3. $(2x^2 + 3x)^x$

   ANS: $(2x^2 + 3x)^x[\ln(2x^2 + 3x) + \frac{(4x+3)x}{2x^2+3x}]$

4. $(x^3 - 1)^4 \ln(x)^2$

   ANS: $12x^2(x^3 - 1)^3 \ln(x)^2 + (x^3 - 1)^4 2\frac{\ln(x)}{x}$

5. $[e^{-3x} + \sin(x)]^{12}$

   ANS: $[e^{-3x} + \sin(x)]^{11}[-3e^{-3x} + \cos(x)]$

6. Find dy/dx if $x^3y + 7xy - x^3 + y^3 = 7$. (Hint: implicit differentiation.)

   ANS: $y' = (-3x^2y - 7y + 3x^2)/(x^3 + 7x + 3y^2)$. Diff to get $3x^2y + x^3y' + 7xy' + 7y - 3x^2 + 3y^2y' = 0$. Regroup terms to get $x^3y' + 7xy' + 3y^2y' = -3x^2y - 7y + 3x^2$. Solve for y' to get answer.

7. What is the equation of the tangent line (also called: linear approximation) to the curve $y = x^3 + 10x + \ln(x^2 + 7)$ at $x = 1$?
   [You would not be given this **hint** on the real final, but: a method is (1) find the slope s of this curve at $x = 1$ by finding an equation for y' then substituting $x = 1$ into it, (2) find the value v of y at $x = 1$, (3) use the fact that the equation of a line with slope s passing through a point $(x = B, y = v)$ is $y_{\text{line}} = (x - B)s + v$, (4) write down this line-equation in our case! (5) perhaps try a sanity check.]

   ANS: $y_{\text{line}} = (x - 1)(13 + 1/8) + [11 + \ln(8)]$

8. If the distance an object has traveled after t seconds is $5t^2 + t^3$ meters, then what is the object's speed when $t = 5$?

   ANS: deriv is $10t + 3t^2$. When $t = 5$ get speed$=50 + 75 = 125$ meters/sec.

9. And what is its acceleration? [Note: speed is the time-derivative of (i.e. rate of change of) distance, and acceleration is the time-derivative of (i.e. rate of change of) speed. You would be expected to know those definitions on the real final without being given such a hint.]

   ANS: Second deriv is $10 + 6t$. When $t = 5$ get accel$= 10 + 30 = 40$ meters/sec^2.

10. Find these limits [Show all work. Trying to estimate a numerical value with a calculator is a fine and recommended sanity check, but they will want you to show your work and methods and will give such an approach a lot less credit than if you get the exact answer via an exact method.] $\lim_{x\to 3} \frac{x^2-x-6}{\sin(\pi x)}$

    ANS: Get 0/0 if try to put $x = 3$ directly. So use L'Hopital to reduce to $\lim_{x\to 3}(2x - 1)/[\pi\cos(\pi x)]$ and then $x \to 3$ yields $-5/\pi$.

11. $\lim_{x\to 0} \frac{x^2+5}{x^2}$

    ANS: ∞. (But: $\lim_{x\to 0} \frac{x^2+5}{x}$ would have been "does not exist" since sign of ∞ then would depend on which direction you approach 0 from.)

12. $\lim_{h\to 0} \frac{\tan(x+h)-\tan(x)}{h}$

ANS: $\tan'(x) = \sec(x)^2$. This is the limit-definition of a derivative. [Also one could do this via L'Hopital rule.]

13. Draw a plot of a function $f(x)$ with all the following properties simultaneously:
(a) If $0 < x < 1$ then $f'(x) = 1$.
(b) If $1 < x < 2$ then $f'(x) = -2$.
(c) $f(0) = 0$.
(d) If $2 < x < 4$ then $f''(x)$ is positive.
(e) If $4 < x < 6$ then $f''(x)$ is negative.
(f) $\lim_{x\to\infty} f(x) = 1$.
(g) $f'(6) = 0$.
(h) $f(x)$ is continuous.

ANS (in words): $f(x)$ starts out at $f(0) = 0$ and goes up as a line segment at slop e to reach $f(1) = 1$. Thena corner and goes down at slope -2 as a line segment to reach $f(2) = -1$. Then a concave-∪ region to reach $f(4)$ which is inflection point, then a concave-∩ region to reach $f(6)$ which is a MAXIMUM (flat spot), then continue to asymptote to height 1 as $x \to \infty$.

14. A plane flies west 1000 miles at speed $V + W$ miles per hour, then flies back east at speed $V - W$, because there is a wind W.
(a) What is the total time of the two flights?
(b) What value of W minimizes this time? (show work - numerical sanity checks are a good idea but not enough to get full credit).

ANS: Assume $|W| < V$ since otherwise plane will never get there (blown backward or stays still). Total time is $1000/(V - W) + 1000/(V + W)$. Minimize by taking derivative with respect to W to get $1000/(V - W)^2 - 1000/(V + W)^2$ then set this equal to 0 to get the critical W. Well if $W = 0$ this is 0 since $1000/(V)^2 - 1000/(V)^2 - 0$ and no other W work since one term always bigger than other if $W \neq 0$. So the min is at $W = 0$ and then time=$2000/V$.

15. Consider the function $y(x) = x + 3/x$ for $x > 0$. Find all critical points of this function and find its global minimum and global maximum (give both the x and y values at all of them).

ANS: Derivative $1 - 3/x^2 = 0$ when $x = \pm\sqrt{3}$ and since $x > 0$ was said, only $x = \sqrt{3}$ is valid critical point. This is the min and $y = 2\sqrt{3}$ there. The max is $y = \infty$ as $x \to \infty$ or $x \to 0+$.

16. Suppose each year your salary increases by a factor of 1.03. How many years before your salary is 10 times larger than it is now? (Show work.) [You will not be given this **hint** on the real final, but note that your salary after n years is expressible as a function of n using an exponential. Once you've found this formula, you can solve it= 10 for n by using properties of logarithms.]

ANS: $\ln(10)/\ln(1.03) \approx 77.9$. We know $1.03^n = 10$ and the problem is to find n. Take ln of both sides to get $n \ln 1.03 = \ln 10$ so $n = 77.9$ as above.

17. Find a function $F(x)$ such that $F'(x) = 7x^3 + 9$ and such that $F(0) = 93$. [You will not be given this **hint** on the real final, but note that if you find $F(x)$ satisfying the first condition, about F', then adding any constant to that F *still* will satisfy the first condition. So what value of the constant makes F also satisfy the second condition?]

ANS: $F(x) = (7/4)x^4 + 9x + 93$.