↑name↑ ↑ID#↑ October 30, 2002. Show all work.

Math 75 – Fall 2002 – Quiz 2#2 – Warren D. Smith

1. Find a formula for the derivative of the following functions of x: $\sin(x^3)$

   ANSWER: $\cos(x^3) \cdot 3x^2$ by chain rule

2. $\sin\sqrt{x+7}$.

   ANSWER: $\cos(\sqrt{x+7}) \cdot \frac{1}{2}(x+7)^{-1/2}$ by chain rule (and power rule with power= 1/2).

3. $\frac{\cos(x)}{x-3}$.

   ANSWER: $\frac{-\sin(x)(x-3)-\cos(x)}{(x-3)^2}$ by quotient rule.

4. $x^2\exp(\sin(x))$.

   ANSWER: $2x\exp(\sin(x)) + x^2\exp(\sin(x))\cos(x)$ by product rule then chain rule.

5. $\log_3(2-x)$.

   ANSWER: $\frac{1}{(x-2)\ln 3}$ by first re-expressing as $\ln(2-x)/\ln 3$ by log base-change formula, then differentiate (using chain rule to get the $-$ sign).

6. Find the values of the following limits: $\lim_{x\to 3}\frac{\sin(\pi x)}{x^2+6}$

   ANSWER: just 0 since $\sim(3\pi) = 0$. Easy.

7. $\lim_{x\to 3}\frac{\sin(\pi x)}{x^2-9}$

   ANSWER: just substituting $x = 3$ won't work since would get $0/0$. That tells us to use L'Hopital: get $\frac{\pi\cos(3\pi)}{2\cdot 3} = -\pi/6$.

8. $\lim_{x\to 0}\frac{\ln(x+1)}{\sin(x)}$

   ANSWER: just substituting $x = 0$ won't work since would get $0/0$. That tells us to use L'Hopital: get $\lim_{x\to 0}\frac{1}{(x+1)\cos x} = \frac{1}{(0+1)\cos(0)} = 1/1 = 1$.

9. For $y = x\ln(x)$ on $0 < x < \infty$, find the (x, y) coordinates of all of the following:
   the root (or roots), i.e. find x at which $y = 0$.

   ANSWER: $y = 0$ when $x = 0$ or when $\ln x = 0$. The former is forbidden since question demanded $x > 0$. The latter is when $x = 1 = e^0$.

10. All local mins and local maxes (if any)

    ANSWER: $y' = \frac{x}{x} + \ln x = 1 + \ln x$ is zero (critical point) if and only if $\ln x = -1$ i.e. when $x = e^{-1}$. This x yields $y = -e^{-1}$ which is a *min*. Are no local maxes.

11. All inflection points (if any)

    ANSWER: $y'' = 1/x$. This is *never* 0 when $0 < x < \infty$ so there are *no* inflection points.

12. For which x is this curve concave-∪? For which is it concave-∩?

    ANSWER: it is always concave-∪ since $y'' = 1/x > 0$ always.

13. Find the equation of the tangent line (linear approximation) to the curve $y = x^2 + 7$ at $x = 3$.

    ANSWER: $y_{\text{lin.approx}} = y'(B)(x - B) + y(B)$ where here the basepoint is $B = 3$, and $y'(x) = 2x$. So $y(3) = 3^2 + 7 = 16$ and $y'(3) = 2 \cdot 3 = 6$ so $y_{\text{lin.approx}} = 6(x - 3) + 16$.

14. For $y = \exp(-x^2)$ (for all real x), find the following: the horizontal asymptotes (what y tends to as $x \to +\infty$ and as $x \to -\infty$)

    ANSWER: as $x \to \pm\infty$, $x^2 \to +\infty$, $\exp(-x^2) \to 0$. (Very large negative power of e, so huge denominator...) So answer $y = 0$.

15. (x, y) coordinates of all local maxes and mins (if any)

    ANSWER: $y'(x) = -2x\exp(-x^2)$ (got by the chain rule) is 0 only when $x = 0$ (since e^{anything} is never 0). So this ($x = 0$, $y = 1$) is the only critical point, which is a *max*.

16. (x, y) coordinates of inflection points (if any)

    $y'' = 4x^2\exp(-x^2) - 2\exp(-x^2)$ by the product rule. This is $y'' = (4x^2 - 2)\exp(-x^2)$. This is 0 if $4x^2 - 2 = 0$ i.e. if $2x^2 = 1$ i.e. if $x = \pm\sqrt{1/2}$ (and then $y = \exp(-1/2)$. These are the coords of the inflection points.

17. For which x is this curve concave-$\cap$?

    If $-\sqrt{1/2} < x < \sqrt{1/2}$,. i.e. if x lies between the inflection points.

18. sketch a plot of the derivative of the function pictured below: