

## MATH 163 PROBLEM SET 1

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ABSTRACT. Due Monday, January 30, 2006

**Problem 1.** Prove by induction that there are at most  $n/2$  odd numbers  $x$  such that  $1 \leq x \leq n$ .

**Problem 2 – Pigeonhole Principle.** Prove by induction that if you have  $n$  letters and  $n - 1$  mailboxes than at least one mailbox has at least 2 letters in it.

**Problem 3.** Prove by induction that for every pair of consecutive numbers  $n, n + 1$  at least one of the two numbers is odd.

**Problem 4.** Consider the sequence  $a_1, a_2, \dots, a_n, \dots$ , where  $a_1 = 1$ , and  $a_{n+1} = 2a_n + 1$ , for every  $n$ . Prove that  $a_k = 2^k - 1$ .

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*Key words and phrases.* discrete, mathematics, sets, logic, combinatorics, graphs.