MSRC – Math 85 Review for Final – Spring 2006

1. The graph of f is given.

a) Find each limit, or explain why it does not exist.

(i) $\lim_{x \to 2^+} f(x)$ (ii) $\lim_{x \to -3^+} f(x)$ (iii) $\lim_{x \to -3} f(x)$

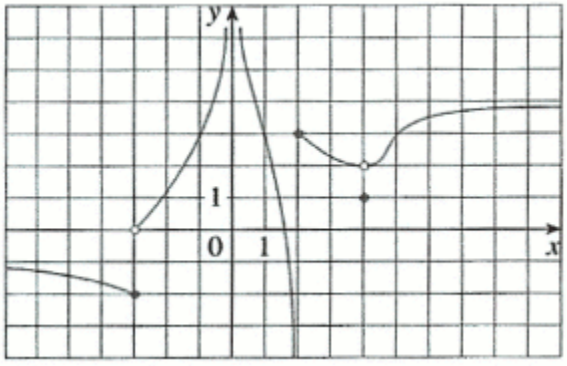
(iv) $\lim_{x \to 4} f(x)$ (v) $\lim_{x \to 0} f(x)$ (vi) $\lim_{x \to 2^-} f(x)$

(vii) $\lim_{x \to \infty} f(x)$ (viii) $\lim_{x \to -\infty} f(x)$

b) State the equations of the horizontal asymptotes.

c) State the equations of the vertical asymptotes.

d) At what numbers is f discontinuous? Explain. Classify each discontinuity as removable, jump, or infinite.



2. Sketch the graph of an example of a function f that satisfies all of the following conditions:

$\lim_{x \to 0^+} f(x) = -2$ $\lim_{x \to 0^-} f(x) = 1$ $f(0) = -1$

$\lim_{x \to 2^-} f(x) = \infty$ $\lim_{x \to 2^+} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = 3$ $\lim_{x \to -\infty} f(x) = 4$

3. Let $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \le x < 3 \\ (x-3)^2 & \text{if } x > 3 \end{cases}$

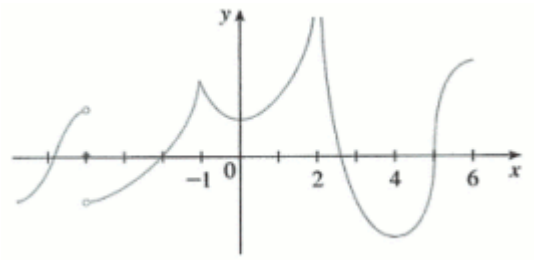
a) Evaluate each limit, if it exists.

(i) $\lim_{x \to 0^+} f(x)$ (ii) $\lim_{x \to 0^-} f(x)$ (iii) $\lim_{x \to 0} f(x)$

(iv) $\lim_{x \to 3^-} f(x)$ (v) $\lim_{x \to 3^+} f(x)$ (vi) $\lim_{x \to 3} f(x)$

b) Where is f discontinuous?

4. The graph of f is shown. State, with reasons, the numbers at which f is not differentiable.



5. Let $f(x)=\begin{cases} x^2 & \text{if } x<0 \\ e^x+2cx-1 & \text{if } x\geq 0 \end{cases}$

a) For what values of c is f continuous at 0?
b) Find all numbers c that make f differentiable at 0.

6. Let $f(x)=\begin{cases} 2-x & \text{if } x\leq 1 \\ x^2-2x+2 & \text{if } x>1 \end{cases}$

a) Is f continuous at 1?
b) Is f differentiable at 1?

7. Let $f(x)=\begin{cases} x^2 & \text{if } x\leq 2 \\ mx+b & \text{if } x>2 \end{cases}$ Find the values of m and b that make f differentiable.

In 8 – 19, find the limit.

8. $\lim\limits_{x\to -3}\dfrac{x^2-9}{x^2+2x-3}$ **9.** $\lim\limits_{x\to 8^-}\dfrac{|x-8|}{x-8}$ **10.** $\lim\limits_{x\to 7}\dfrac{\sqrt{x+2}-3}{x-7}$ **11.** $\lim\limits_{x\to\infty}\dfrac{1+2x-x^2}{1-x+2x^2}$

12. $\lim\limits_{x\to\infty} e^{-3x}$ **13.** $\lim\limits_{x\to 1}\dfrac{2-x}{(x-1)^2}$ **14.** $\lim\limits_{x\to 0^-}\csc x$ **15.** $\lim\limits_{x\to 0}\dfrac{\tan \pi x}{\ln(1+x)}$

16. $\lim\limits_{x\to\infty}\dfrac{e^{4x}-1-4x}{x^2}$ **17.** $\lim\limits_{x\to 0}\dfrac{x+\sin x}{x+\cos x}$ **18.** $\lim\limits_{x\to 0}(1-2x)^{1/x}$ **19.** $\lim\limits_{x\to 0^+}(\cos x)^{1/x^2}$

In 20 – 29, calculate y'.

20. $y=\sqrt{x}+\dfrac{1}{\sqrt[3]{x^4}}$ **21.** $y=\dfrac{e^x}{1+x^2}$ **22.** $y=\arcsin(e^x)$ **23.** $y=\tan\sqrt{1-x}$

24. $y=xe^{-1/x}$ **25.** $y=e^{\cos x}+\cos(e^x)$ **26.** $y=\ln(x^2e^x)$ **27.** $y=\sec(1+x^2)$

28. $xy^4+x^2y=x+3y$ **29.** $\sin(xy)=x^2-y$

30. If $f(t)=\sqrt{4t+1}$, find $f''(2)$.

31. If $g(\theta)=\theta\sin\theta$, find $g''(\pi/6)$

Find an equation of the tangent to the curve at the given point.

32. $y=4\sin^2 x,\quad \left(\frac{\pi}{6},1\right)$

33. $x^2+4xy+y^2=13,\ (2,1)$

34. Suppose that $h(x)=f(x)g(x)$ and $F(x)=f(g(x))$, where $f(2)=3,\ g(2)=5$, $g'(2)=4,\ f'(2)=-2$ and $f'(5)=11$. Find (a) $h'(2)$ and (b) $F'(2)$.

35. If $f(x)=g(x^2)$, find f' in terms of g' .

In 36 - 38, use differentials (or equivalently, a linear approximation) to estimate the given number.

36. $\sqrt{99.8}$ **37.** $(8.06)^{2/3}$ **38.** $1/1002$

39. A particle moves according to a law of motion $s=s(t),\ t\geq 0$, where *t* is measured in seconds and *s* in meters.

a) Express $v(t)$ and $a(t)$.

b) Find $v(3)$ and $a(3)$ and determine whether the particle is speeding up or slowing down after 3 seconds.

c) When does the particle change the direction of motion?

(i) $s(t)=t^3-12t^2+36t$ **(ii)** $s(t)=\dfrac{t}{t^2+1}$

40. Find the local and absolute extreme values of the function on the given interval

(i) $f(x)=x-\sqrt{x},\ [0,4]$

(ii) $f(x)=\dfrac{\ln x}{x^2},\ [1,3]$

In 41 – 42, sketch the graph of a function that satisfies all of the given conditions

41. $f(0)=0$, $f'(-2)=f'(1)=f'(9)=0$,
$\lim_{x\to\infty} f(x)=0$, $\lim_{x\to 6} f(x)=-\infty$,
$f'(x)<0$ on $(-\infty,-2)$, (1, 6), and $(9,\infty)$
$f'(x)>0$ on $(-2,1)$ and (6, 9),
$f''(x)>0$ on $(-\infty,0)$ and $(12,\infty)$,
$f''(x)<0$ on (0, 6) and (6, 12).

42. $f'(x)>0$ if $|x|<2$, $f'(x)<0$ if $|x|>2$,
$f'(-2)=0$, $\lim_{x\to 2}|f'(x)|=\infty$, $f''(x)>0$ if $x\neq 2$.

43. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

44. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

45. Find the point on the line $y=4x+7$ that is closest to the origin.

46. Find the dimensions of the rectangle of largest area that has two of its sides on the coordinate axes and one of its vertices on the parabola $y=4-x^2$ in the first quadrant.

Find the function f :

47. $f'(x)=8x-3\sec^2 x$

48. $f'(x)=e^x-(2/\sqrt{x})$

49. $f'(t)=2t-3\sin t, \quad f(0)=5$

50. $f'(x)=\dfrac{4}{\sqrt{1-x^2}}, \quad f(1/2)=1$