Math. 86: Exam #2

1. (48 points) Evaluate the integrals

$$(a)\ \int \sin^3 x \cos^2 x \, dx \qquad (b)\ \int_0^{\pi/6} \cos^2(3x)\, dx \qquad (c)\ \int \tan^3 x \sec^3 x \, dx$$

$$(d)\ \int_0^{1/2} \frac{x^2}{(1-x^2)^{3/2}}\, dx \qquad (e)\ \int \frac{x^2}{x^2-2x+3}\, dx \qquad (e)\ \int \frac{3x^2+15x-18}{(x+3)(x^2+9)}\, dx$$

2. (12 points) Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$(a)\ \int_0^{\infty} xe^{-x^2}\, dx \qquad (b)\ \int_{-1}^{1} \frac{1}{x^3} dx$$

3. (6 points) Use the Comparison Theorem to determine whether the integral $\int_1^{\infty} \frac{1}{x^3+x+1}\, dx$ is convergent of divergent.

4. (12 points) (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{3(-2)^{n+1}}{3^n}$ converges or diverges. If it converges, find its sum.

(b) Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n}$ converges. Find the sum of the series for those values of x.

5. (30 points) Determine whether the following series converge or diverge. Be sure to indicate what test you are using and carry out all work related to that test.

$$(a)\ \sum_{n=1}^{\infty} ne^{-n^2} \qquad (b)\ \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2+1} \qquad (c)\ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n-1}{n+2\sqrt{n}}$$

$$(d)\ \sum_{n=1}^{\infty} \frac{4^n+1}{3^n-1} \qquad (e)\ \sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{n^5+5}}$$

Total: 108 points

$$\sin^2 x + \cos^2 x = 1 \qquad \cos^2 x = \frac{1 + \cos\, 2x}{2} \qquad \sin^2 x = \frac{1 - \cos\, 2x}{2} \qquad 1 + \tan^2 x = \sec^2 x$$

$$\int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x\, dx = \sec x + C \qquad \int \tan x\, dx = ln|\sec x| + C$$

$$\int \sec x\, dx = ln|\sec x + \tan x| + C$$