

SOLUTION TO A SERIES PROBLEM POSTED BY HISSAB KEBEDEW

The problem is to find the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)}$$

Recall from integration techniques the method of partial fractions which involves writing a function like $\frac{1}{x(x+1)}$ as a sum of two terms of the form $\frac{A}{x} + \frac{B}{x+1}$. Try to use this as a hint and go back and think about the series again before reading what follows.

If you are stuck, here is how one can solve it:

Consider first the product of the first two factors $\frac{1}{k(k+1)}$ which one can write as :

$$\frac{1}{k(k+1)} = \frac{A}{k} + \frac{B}{k+1}$$

with $A = 1$ and $B = -1$. (Just review the method of partial fractions). Using this, we can now write the term to be summed up as:

$$\frac{1}{k(k+1)(k+2)} = \frac{1}{k(k+2)} - \frac{1}{(k+1)(k+2)}$$

Observe that we can again apply the method of partial fractions to each of the two terms on the right hand side to get:

$$\frac{1}{k(k+2)} = \frac{1}{2} \left(\frac{1}{k} - \frac{1}{k+2} \right)$$

and likewise

$$\frac{1}{(k+1)(k+2)} = \frac{1}{k+1} - \frac{1}{k+2}$$

(notice that this latter formula also follows easily from the one for $\frac{1}{k(k+1)}$). Finally, note that what you have to sum up are two series each of which telescopes. These details are left to the student.

Concerning a reference for Math 262, I suggest that you look at different calculus books in addition to your textbook and pick one or two that you like. Sometimes you may understand a concept better after you read the expositions in several books. Authors differ in the way they present mathematical concepts just like teachers differ in their lecturing styles. You will also find more worked out examples when you look at different books.