

## Corrections and updates for “Multiplicative Invariant Theory”

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**page 18, line 6:**  $G$  and  $H$  are interchanged. It should read “Then the  $G$ -module  $\mathbb{Z}[G] \otimes_{\mathbb{Z}[H]} M$  is in fact a  $G$ -lattice; ...” (Thanks to Robert Fossum.)

**page 21, line -1:** A backslash is missing in front of “*emph*”. It should be “The group  $G$  is called *k-reflection group* on  $V$  if  $G = \mathcal{R}_V^k(G)$ .” (Thanks to Robert Fossum.)

**page 25, axiom (B2):** The formula  $\Phi = \mathbb{Z}_+ \Delta \coprod -\mathbb{Z}_+ \Delta$  should be

$$\Phi \subseteq \mathbb{Z}_+ \Delta \cup -\mathbb{Z}_+ \Delta .$$

**page 103, line -1:** “*k-reflection*” should be replaced by “*bireflection*”; so

$$\mathcal{R}^2(\mathcal{H}) = \langle g \in \mathcal{H} \mid g \text{ acts as a bireflection on } L \rangle .$$

**page 126, line 8 and page 127, line 4:** The list of “primes”  $p = 47, 112, 223, \dots$  is wrong as stated; it should be  $p = 47, 113, 233, \dots$ . Thus, on page 127, it should read “In particular, the aforementioned non-rational extensions  $\mathbb{Q}(\mathcal{C}_p)/\mathbb{Q}$  ( $p = 47, 113, 233, \dots$ ) are not stably rational either.” (Thanks to Don Passman.)

**page 126, lines -9 and -12:**  $K$  should be replaced by  $\mathbb{k}$  twice: it should read “... Saltman [175] for infinite  $\mathbb{k}$  ...” and “...  $E/F$  of  $\mathbb{k}$  with group  $\mathcal{G}$  ...”.

**page 149, line -4:** The answer to Problem 2, when stated more generally for  $k$ -reflections, is definitely negative. Alex Zalesskiĭ has shown me the following example. The simple group  $\mathrm{SL}_2(\mathbb{F}_{32})$  has an irreducible Steinberg module,  $V$ , of dimension 32 over  $\mathbb{Q}$ . The restriction of  $V$  to the Sylow 2-subgroup of  $\mathrm{SL}_2(\mathbb{F}_{32})$  is the regular module. Therefore, each involution of  $\mathrm{SL}_2(\mathbb{F}_{32})$  acts as a 16-reflection. Furthermore, the involutions generate  $\mathrm{SL}_2(\mathbb{F}_{32})$ . On the other hand, if  $g \in \mathrm{SL}_2(\mathbb{F}_{32})$  is an element of order 31 then its fixed points on  $V$  have dimension 2 and  $\langle g \rangle$  is an isotropy group on  $V$ . Thus, this isotropy group is only generated by a 30-reflection.

**page 150, line 12:** omit “over  $\mathbb{Z}$ ”.

**page 154, line 5 after Problem 7:** Replace  $|\mathcal{G}|/r$  by  $r/|\mathcal{G}|$  (twice).

**Problem 14 (page 159):** This has been solved in the affirmative; the reference is:

A. Hoshi and Y. Rikuna, *Rationality problem of three-dimensional purely monomial group actions: the last case*, Math. Comp. **77** (2008), 1823–1829.

Now all multiplicative invariant fields of transcendence degree at most 3 are known to be rational over the base field.