Real Analysis II, Prof. Gutiérrez, Absolute continuity
Week of February 5, 2009

1. Show that if f and g are absolutely continuous functions in $[a, b]$ and $f'(x) = g'(x)$ a.e., then $f(x) - g(x) =$ constant, for each $x \in [a, b]$.

2. If f is absolutely continuous in $[0, 1]$, then f^2 is absolutely continuous in $[0, 1]$.

3. There exists $f \in C^\alpha([0, 1])$ for all $0 < \alpha < 1$ such that f is not absolutely continuous on $[0, 1]$. Take for example $f(x) = x \sin \frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$.

4. Let f be a continuous function in $[0, 1]$ such that f is absolutely continuous in $[0, \epsilon]$ for every $\epsilon, 0 < \epsilon < 1$. Show that f is absolutely continuous in $[0, 1]$.

5. Let f_n be absolutely continuous functions in $[a, b]$, $f_n(a) = 0$. Suppose f_n' is a Cauchy sequence in $L^1([a, b])$. Show that there exists f absolutely continuous in $[a, b]$ such that $f_n \to f$ uniformly in $[a, b]$.