

## LAGRANGE MULTIPLIERS, DIFFERENTIALS, VECTOR FIELDS

- Find the maximum product of  $x^2yz$  if  $x, y$ , and  $z$  are positive numbers such that  $3x + 2y + z = 24$ . ANSWER: 288 at  $(4, 3, 6)$ .
- Find the maximum value of  $f(x, y, z) = x + y + 4z$  on the sphere  $x^2 + y^2 + z^2 = 8$ . ANSWER: 12 at  $(2/3, 2/3, 8/3)$ .
- Find the maximum and minimum values of  $x^2 + y^2$  subject to the constraint  $-6x^2 - 3x - 6y^2 + 4y = 0$ . ANSWER: Min 0 at  $(0, 0)$ ; max  $25/36$  at  $(-1/2, 2/3)$ .
- Find the maximum value of  $f(x, y) = 7 - x^2 - y^2$  on the line  $x + 3y = 2$ . ANSWER:  $33/5$  at  $(1/5, 3/5)$ .
- Let  $f(x, y) = x^2y - 2xy + y^2x$ . Find the increment  $\Delta f$  and the differential  $df$  if  $(x, y)$  varies from  $(1, 0)$  to  $(1.1, -0.1)$ . ANSWER: 0.11; 0.1
- If  $f(x, y, z) = \frac{x^2y^4}{1 + z^2}$ , use differentials to approximate  $f(4.996, 1.003, 1.995)$ . ANSWER: 5.072
- Determine whether the following vector fields are the gradient  $\nabla f$  of a function  $f$ .
  - $F(x, y) = (2x + y^3)\mathbf{i} + (3xy^2 - e^{-2y})\mathbf{j}$ .
  - $F(x, y) = (3\cos y + 2\sin x)\mathbf{i} + (3y^2 - 3x\sin y)\mathbf{j}$ .

ANSWER: (a)  $f(x, y) = x^2 + xy^3 + \frac{1}{2}e^{-2y} + C$ ; (b)  $f(x, y) = 3x\cos y - 2\cos x + y^3 + C$ .

## MULTIPLE INTEGRATION

- Evaluate  $\int_0^1 \int_{-2}^0 (2x^2y^2 - 2xy^2) dydx$ . ANSWER:  $-8/9$
- Integrate the function  $f(x, y) = 6x + 4y$  over the triangular region cut from the first quadrant by the line  $x + y = 2$ . ANSWER:  $40/3$
- Find the area of the region bounded by  $x = 4y^2$  and  $x = 5y - y^2$ . ANSWER:  $5/6$
- Find the area of the region bounded by the curves  $y = x^4$  and  $y = 4 - 3x^2$ . ANSWER:  $28/5$
- Find the value of  $\int_0^{10/3} \int_1^{3-\frac{3}{5}x} dydx$  by evaluating an equivalent integral with the order of integration reversed.  
ANSWER:  $\int_1^3 \int_0^{5-\frac{5}{3}y} dx dy = 10/3$ .
- Find the area enclosed by one leaf of the rose  $r = 5\cos(6\theta)$ . ANSWER:  $25\pi/24$
- Change  $\int_0^6 \int_0^{\sqrt{9-(x-3)^2}} \frac{x+y}{x^2+y^2} dydx$  to an equivalent polar integral and evaluate the polar

integral. ANSWER:  $3 + \frac{3}{2}\pi$

**8.** Find the volume of the region enclosed by the cylinder  $y^2 + 9z^2 = 36$  and the planes  $x = 0, x + y = 6$ . ANSWER:  $72\pi$

**9.** True or false: The value of  $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^{-3x+y} dz dx dy$  is  $2/3$ . ANSWER: true

**10.** Find the volume of the solid in the first octant bounded by the coordinate planes, the plane  $x = 6$  and the parabolic cylinder  $z = 4 - y^2$ . ANSWER: 32

**11.** Write six different iterated triple integrals for the volume of the tetrahedron cut from the first octant by the plane  $4x + 12y + 3z = 12$ .

**12.** Find the volume of the region in the first octant bounded by the planes  $x + z = 6$  and  $y + 2z = 12$ . ANSWER: 144

**13.** Find the volume of the region bounded above by the paraboloid  $z = 9 - x^2 - y^2$ , below the  $xy$ -plane, and that lies outside the cylinder  $x^2 + y^2 = 4$ . ANSWER:  $25\pi/2$

**14.** Find the smaller volume cut from the sphere  $\rho = 2$  by the plane  $z = 1$ . ANSWER:  $5\pi/3$

**15.** Find the volume of the region cut from the solid sphere  $x^2 + y^2 + z^2 = 100$  by the cylinder  $x^2 + y^2 = 25$ . ANSWER:  $\frac{4\pi}{3}(1000 - 75^{3/2}) \approx 1468.09$

**16.** Consider the region  $R$  bounded by  $x + y = 1, x + y = 2, 2x - 3y = 2$ , and  $2x - 3y = 5$ .

(a) Find the image the transformation  $u = x + y$  and  $v = 2x - 3y$  of the region  $R$ . Sketch the transformed region in the  $uv$ -plane.

(b) Show that the inverse transformation is given by  $x = \frac{3u + v}{5}, y = \frac{2u - v}{5}$  and use the formula of change of variables to evaluate  $\int_R x dA$ .

ANSWER:  $1 \leq u \leq 2; 2 \leq v \leq 5; \frac{\partial(x, y)}{\partial(u, v)} = -\frac{1}{5}; \int_R x dA = \int_1^2 \int_2^5 \frac{3u + v}{5} \frac{1}{5} dv du = 24/25$ .

**17.** Evaluate the integral  $\int_C (2y^2x + 3y^2) dx + (2x^2y + 6yx) dy$  where  $C$  is the straight segment from  $(1, 1)$  to  $(2, 3)$ . ANSWER: 86.

**18.** Evaluate the integral of  $f(x, y, z) = -2x - 2y + z + 2$  along the curve  $\mathbf{r}(t) = t\mathbf{i} + (-2 + 2t)\mathbf{j} + 3\mathbf{k}$ ,  $0 \leq t \leq 1$ . ANSWER:  $6\sqrt{5}$ .

**19.** Find the work done by the force  $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$  from  $(0, 0, 0)$  to  $(1, 1, 1)$  along the curve  $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}$ ,  $0 \leq t \leq 1$ . ANSWER:  $69/70$ .