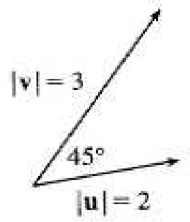
Math 127 **MSRC Review for FINAL EXAM** **Fall 2006**

Chapter 12

Review Exercises

3. If $\mathbf{u}$ and $\mathbf{v}$ are the vectors shown in the figure, find $\mathbf{u} \cdot \mathbf{v}$ and $|\,\mathbf{u} \times \mathbf{v}|$.



11. (mod.) (**a**) Find an equation of the plane through the points $A\,(1, 0, 0)$, $B\,(2, 0, -1)$, and $C\,(1, 4, 3)$.
(**b**) Find the area of triangle ABC.

15. Find parametric equations for the line through $(4, -1, 2)$ and $(1, 1, 5)$.

17. Find parametric equations for the line through $(-2, 2, 4)$ and perpendicular to the plane $2x - y + 5z = 12$.

18. Find an equation of the plane through $(2, 1, 0)$ and parallel to the plane $x + 4y - 3z = 1$.

Chapter 13

Review Exercises

8. Find the length of the curve $\mathbf{r}(t) = \left\langle 2t^{3/2},\, \cos 2t,\, \sin 2t \right\rangle, \quad 0 \le t \le 1.$

18. A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\,\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\,\mathbf{i} + 12t^2\,\mathbf{j} - 6t\,\mathbf{k}$. Find its position function.

Chapter 14

Review Exercises

33. Find the linearization $L(x, y, z)$ of the function $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at the point $(2, 3, 4)$ and use it to estimate the number $(1.98)^3\sqrt{(3.01)^2 + (3.97)^2}$.

36. If $z = \cos xy + y \cos x$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$.

43. Find the gradient of the function $f(x, y, z) = z^2\, e^{x\sqrt{y}}$.

46. Find the directional derivative of $f(x, y, z) = x^2 y + x\sqrt{1 + z}$ at the point $(1, 2, 3)$ in the direction of $\mathbf{v} = 2\,\mathbf{i} + \mathbf{j} - 2\,\mathbf{k}$.

54. Find the local maximum and minimum values and saddle points of the function $f(x, y) = (x^2 + y)\, e^{y/2}$.

Chapter 15

15.8

11. Use cylindrical coordinates to evaluate $\iiint_E x^2\,dv$, where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$.

Review Exercises

7. Calculate the iterated integral $\displaystyle\int_0^{\pi}\int_0^1\int_0^{\sqrt{1-y^2}} y\sin x\,dz\,dy\,dx.$

14. Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1\int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3}\,dx\,dy$$

42. Use spherical coordinates to evaluate $\displaystyle\int_{-2}^{2}\int_0^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dz\,dx\,dy.$

Chapter 16

Review Exercises

6. Evaluate the line integral $\int_C \sqrt{xy}\,dx+e^y\,dy+xz\,dz$, where C is given by

$\mathbf{r}(t)=t^4\,\mathbf{i}+t^2\,\mathbf{j}+t^3\,\mathbf{k},\ \ 0\le t\le 1.$

16.5

18. Determine whether or not the vector field $\mathbf{F}(x,y,z)=y\cos(xy)\,\mathbf{i}+x\cos(xy)\,\mathbf{j}-\sin z\,\mathbf{k}$ is conservative. If it is conservative, find a function f such that $\mathbf{F}=\nabla f$.

Review Exercises

14. Show that $\mathbf{F}(x,y,z)=e^y\,\mathbf{i}+(xe^y+e^z)\,\mathbf{j}+ye^z\,\mathbf{k}$ is conservative and use this fact to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ along the line segment C from $(0,2,0)$ to $(4,0,3)$.

16. Use Green's Theorem to evaluate $\displaystyle\int_C \sqrt{1+x^3}\,dx+2xy\,dy$, where C is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,3)$, positively oriented.

18. Find $\operatorname{curl}\mathbf{F}$ and $\operatorname{div}\mathbf{F}$ if

$\mathbf{F}(x,y,z)=e^{-x}\sin y\,\mathbf{i}+e^{-y}\sin z\,\mathbf{j}+e^{-z}\sin x\,\mathbf{k}$

19. Show that there is no vector field $\mathbf{G}$ such that

$\operatorname{curl}\mathbf{G}=2x\,\mathbf{i}+3yz\,\mathbf{j}-xz^2\,\mathbf{k}$