

FINAL EXAM

FALL 2007

December 11, 2007

Show your work, the right solution without explanation is useless. Be clean and organized, it is your responsibility to make yourself understood. No graphic calculators. If you did not bring your calculator, you will have to do the exam without it, sharing calculators is not allowed. Good luck!!!

| Question | Points | Out of |
|----------|--------|--------|
| 1        |        | 8      |
| 2        |        | 12     |
| 3        |        | 8      |
| 4        |        | 10     |
| 5        |        | 12     |
| 6        |        | 11     |
| 7        |        | 12     |
| 8        |        | 12     |
| 9        |        | 10     |
| 10       |        | 10     |
| Total    |        | 105    |

1. (8 points) Sketch the graph of a function on the interval  $[-2,6]$  with the following properties.

|   |  |
|---|--|
| <p>a. <math>f''(x) &gt; 0</math>, for <math>-2 &lt; x &lt; 0</math> and for <math>2 &lt; x &lt; 4</math></p> <p>b. <math>f''(x) &lt; 0</math>, for <math>0 &lt; x &lt; 2</math> and for <math>4 &lt; x &lt; 6</math></p> <p>c. <math>f'(x) &lt; 0</math>, for <math>x &lt; -1</math> and for <math>x &gt; 5</math></p> <p>d. <math>f'(x) &gt; 0</math>, for <math>-1 &lt; x &lt; 5</math></p> <p>e. <math>f(-1) = -2</math> and <math>f(5) = 4</math></p> |  |
|---|--|

2. (3 points) Use the rules for differentiation to find the derivative of each of the given functions. Do not simplify.

a.  $f(x) = 8\sqrt{x^5} - \sqrt[3]{\frac{1}{x^{11}}}$

c.  $h(x) = \sqrt{x} \ln(x)$

Answer: \_\_\_\_\_

Answer: \_\_\_\_\_

b.  $g(x) = \cos(\ln x)$

d.  $i(x) = \frac{\sin x}{x^2 + 1}$

Answer: \_\_\_\_\_

Answer: \_\_\_\_\_

3. (8 points) Decide if the following functions could be linear, exponential or neither. If linear or exponential give the possible formula.

| $x$ | $f(x)$ | $g(x)$ | $h(x)$ |
|-----|--------|--------|--------|
| 0   | 3      | 3      | 3      |
| 1   | 1.5    | 1.5    | 1.5    |
| 2   | 0.75   | 0.75   | 0      |
| 3   | 0.375  | 0.375  | -1.5   |
| 4   | 0.225  | 0.1875 | -3     |

Answer: \_\_\_\_\_

4. (2 points each) True or False? Circle one.

a.  $\int_a^b [f(x)g(x)]dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right).$

**TRUE** or **FALSE**

b.  $6 + 6\left(\frac{3}{2}\right) + 6\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right)^3 + \dots + 6\left(\frac{3}{2}\right)^{100}$   
 $= \frac{6}{1-3/2}$

c.  $\int \frac{x^3 - 9x + 5}{3x^2 - 9} dx = \ln|3x^2 - 9| + C$

**TRUE** or **FALSE**

**TRUE** or **FALSE**

d.  $\int_a^b \sqrt{f(x)} dx = \sqrt{\int_a^b f(x) dx}.$

**TRUE** or **FALSE**

e.  $\int e^{x^2} dx = \frac{e^{x^2}}{2x} + C$

**TRUE** or **FALSE**

5. (4 points each) Find the sum, if it exists. If it does not exist explain why.

a.  $3 + 3\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + \dots$

Answer: \_\_\_\_\_

b.  $200 + 400 + 800 + 1600 + \dots + 6400.$

Answer: \_\_\_\_\_

c.  $30 + 30(1.1) + 30(1.1)^2 + 30(1.1)^3 + \dots$

Answer: \_\_\_\_\_

6. Let  $f(x) = 2x^5 - 5x^4 - 10x^3 + 5$
- (2 points) Find the first derivative function  $f'(x)$ , and use it to determine the critical points of  $f(x)$ .

Answer: \_\_\_\_\_

- (3 points) Determine where  $f(x)$  is increasing and decreasing.

Answer: \_\_\_\_\_

- (2 points) Find all local maxima and minima using the First Derivative Test.

Answer: \_\_\_\_\_

- (2 points) Use the Second Derivative Test to decide if the critical points are local maxima or local minima. If this test is inconclusive at some point explain why.

Answer: \_\_\_\_\_

- (2 points) Find all inflection points.

Answer: \_\_\_\_\_

7. (3 points each) Evaluate each of the following definite or indefinite integrals.

a.  $f(x) = \int x^8 + \sqrt{\frac{1}{x^7}} dx.$

c.  $h(x) = \int 6xe^{3x^2} dx.$

Answer: \_\_\_\_\_

Answer: \_\_\_\_\_

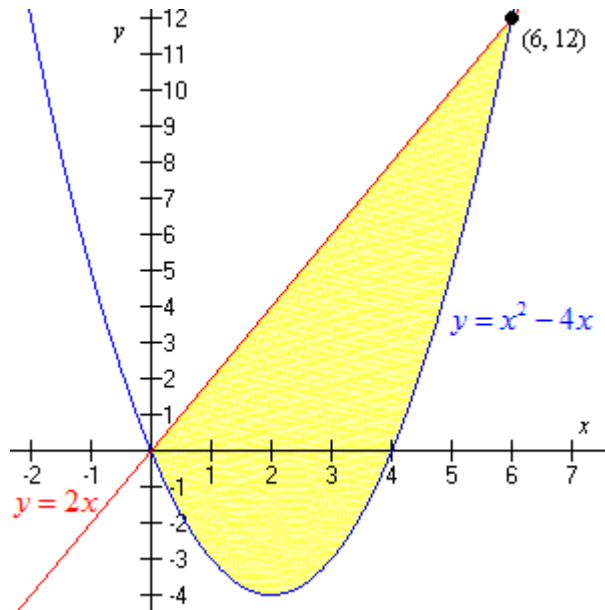
b.  $g(x) = \int \frac{6x}{x^2 - 2} dx.$

d.  $i(x) = \int_1^2 \left(3x - \frac{1}{x}\right)^2 dx.$

Answer: \_\_\_\_\_

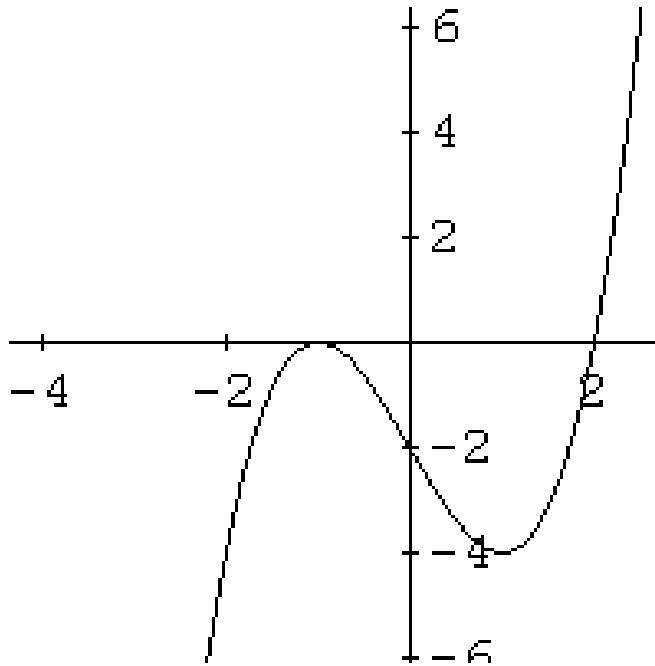
Answer: \_\_\_\_\_

8. (12 points) Graphs of  $f(x) = 2x$  and  $g(x) = x^2 - 4x$  are given below. Use a definite integral to calculate the area enclosed by the graphs of these two functions.



Answer: \_\_\_\_\_

9. The graph of the **derivative** function  $f'(x)$  is given below.



- a. (2 points) Mark in the graph the critical points of  $f(x)$ .
- b. (3 points) Mark in the graph the intervals in which  $f(x)$  is increasing or decreasing.
- c. (2 points) Mark in the graph all local maxima and minima for the function  $f(x)$ .
- d. (3 points) Is  $f(1)$  bigger than  $f(2)$ ? Explain.

Answer: \_\_\_\_\_

10. Let  $f(x) = x^3 + e^{1-x}$

a. (2 points) What is the tangent line of  $f(x)$  at  $x=1$ ?

Answer: \_\_\_\_\_

b. (2 points) Use the tangent line to estimate  $f(0.9)$ .

Answer: \_\_\_\_\_

c. (3 points) Find two antiderivatives of  $f(x)$ .

Answer: \_\_\_\_\_

d. (3 points) What is the average value of  $f(x)$  in the interval  $[1,2]$ .

Answer: \_\_\_\_\_