Instructor_____

MATH C077 **FINAL EXAM** SPRING 2005

Name :

For full credit show all your work

Problem	Point Value	Score
1	6	
2	9	
3	20	
4	6	
5	8	
6	6	
7	6	
8	10	
9	10	
10	12	
11	12	
Total	105	

1. (6*pt*) For each of the functions $f(x)$, $g(x)$, $h(x)$, decide whether it could be linear or exponential. Then fill in the table with two more values of each function based on your decision.

x	$f(x)$	$g(x)$	$h(x)$
0	3	27	20
1	1.5	16	12
2	0.75	5	7.2
3	0.375	-6	4.32
4			
5			

2. (9*pt*) Joe invested \$1000 in a steadily growing mutual fund and, at the same time, his father deposited \$10000 in a bank account. The amounts of money t years after initial investment, $P(t)$ and $Q(t)$, respectively, are given by the formulas: $P(t) = 1000\, e^{0.13t}$ and $Q(t) = 10000\, e^{0.03t}$.

(**a**) What is the rate of growth of Joe's account 3 years after the investment. Give the units.

(**b**) In how many years will the money on Joe's account double? (Round your answer to 1 digit after the point.)

(**c**) How long will it take for Joe's account to get even with the account of his father? (Round your answer to 1 digit after the point.)

3. (20*pt*) Use the differentiation rules to find the derivatives of the following functions.

(**a**) $f(x) = \dfrac{1}{x^3} + x^3$

(**b**) $f(x) = \sqrt{x} \cdot e^x$

(**c**) $f(x) = \dfrac{x^2 + 1}{3x + 1}$

(**d**) $f(x) = \sqrt{3x^5 + \ln x}$

4. (6*pt*) Let $f(x) = \ln x + 2x$.

(**a**) Find the slope of the tangent line to the curve $y = f(x)$ at $x = 1$.

(**b**) Use the tangent line at $x = 1$ to approximate $f(0.99)$.

5. (8*pt*) Find the global maximum and global minimum values of the function $f(x) = \dfrac{9}{x} + x$ on the interval $[1, 5]$.

6. (6pt) Sketch the graph of a function $f(x)$ on the interval $[-2, 8]$ that satisfies all of the following conditions.

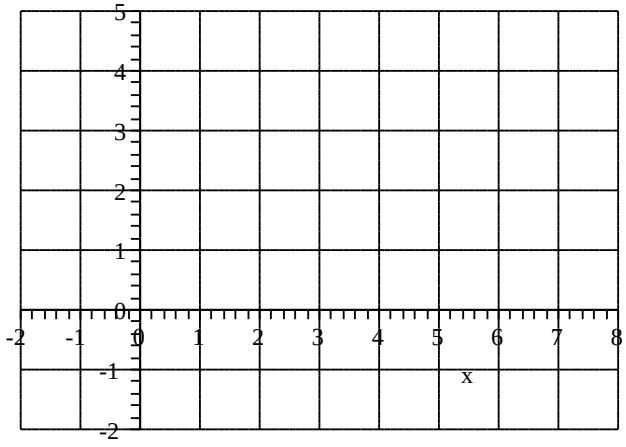
$f(-2) = 4, \quad f(1) = 1, \quad f(3) = 2, \quad f(8) = 4$

$f'(x) < 0$ for $-2 < x < 1$

$f'(x) > 0$ for $1 < x < 8$

$f''(x) > 0$ for $-2 < x < 3$

$f''(x) < 0$ for $3 < x < 8$



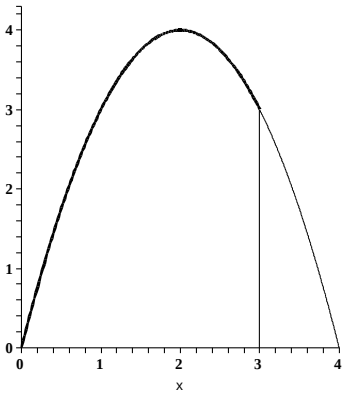
7. (6pt) A car comes to stop 10 seconds after the driver applies the brakes. While the brakes are on, the following velocities are recorded:

Time t (sec)	0	2	4	6	8	10
Velocity $v(t)$ (ft/sec)	100	80	50	25	10	0

(**a**) Give a **lower** estimate of the distance traveled by the car after the brakes were applied.

(**b**) Give an **upper** estimate of the distance traveled by the car after the brakes were applied.

8. (10*pt*) Let $f(x) = 4x - x^2$.

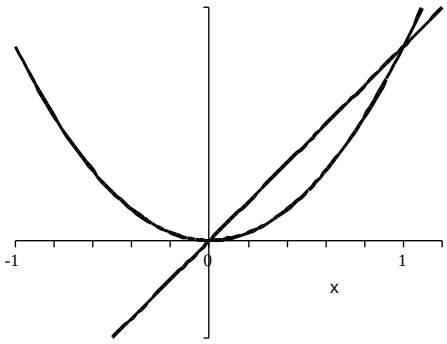


(**a**) Find the average value f_{avg} of the function $f(x)$ on the interval $[0, 3]$.

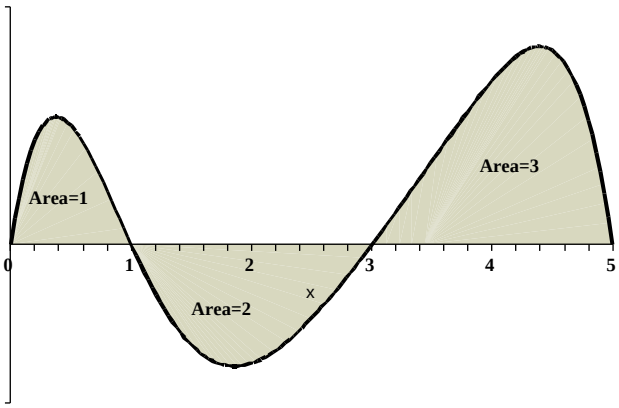
(**b**) Find a value x in the interval $[0, 3]$ for which $f(x) = f_{\text{avg}}$.

(**c**) On the graph of $f(x)$ above, draw the rectangle whose one side is the interval $[0, 3]$ on the x-axis and whose area is equal to the area under the curve $y = f(x)$ between $x = 0$ and $x = 3$.

9. (10*pt*) Find the area of the region enclosed by the curves $y = x^2$ and $y = x$.



10. (12*pt*) The graph of a function $f(x)$ is shown below. Let $F(x)$ be the antiderivative of $f(x)$ such that $F(0) = 1$. Find the following values. Show your work.



(**a**) $\int_0^1 f(x)\,dx =$

(**b**) $\int_0^3 f(x)\,dx =$

(**c**) $\int_0^5 f(x)\,dx =$

(**d**) $F(1) =$

(**e**) $F(3) =$

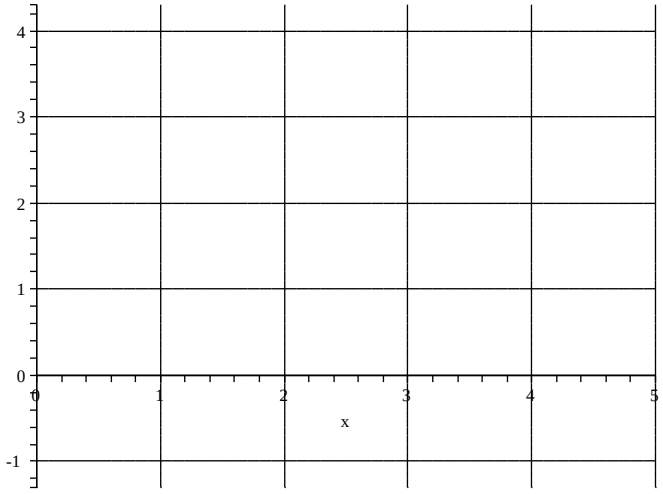
(**f**) $F(5) =$

(**g**) At what number x does $F(x)$ have its global **maximum** on the interval $[0, 5]$?

(**h**) At what number x does $F(x)$ have its global **minimum** on the interval $[0, 5]$?

(**i**) At what numbers x does $F(x)$ have its inflection points on the interval $[0, 5]$?

Use this information to sketch the graph of the function $F(x)$.



11. (12*pt*) Use the method of substitution to evaluate each of the following integrals.

(**a**) $\int 3e^x(e^x+2)^{20}\,dx$

(**b**) $\int_0^1 \frac{2x}{x^2+1}\,dx$