

Math C077 Instructor: \_\_\_\_\_ Your Name: \_\_\_\_\_

**MATH C077 FINAL EXAM**    **SPRING 2007**    **May 3, 2007**

This exam consists of 10 questions.

**SHOW ALL YOUR WORK!    NO WORK, NO CREDIT.**

<b>Question</b>	<b>Points</b>	<b>Out of</b>
<b>1</b>		<b>9</b>
<b>2</b>		<b>12</b>
<b>3</b>		<b>12</b>
<b>4</b>		<b>12</b>
<b>5</b>		<b>12</b>
<b>6</b>		<b>6</b>
<b>7</b>		<b>9</b>
<b>8</b>		<b>9</b>
<b>9</b>		<b>12</b>
<b>10</b>		<b>7</b>
<b>Total</b>		<b>100</b>

1. (3 points each) Given the following function  $f(x)$  :

$x$	0	1
$f(x)$	5	6

- a) If  $f(x)$  is exponential, find the equation for  $f(x)$

Answer: \_\_\_\_\_

- b) If  $f(x)$  is exponential, what is the half-life?

Answer: \_\_\_\_\_

- c) If  $f(x)$  is linear, find the equation for  $f(x)$

Answer: \_\_\_\_\_

2. (3 points each) Let  $f(x) = 8x^2 - 3x$ .
- (a) Estimate  $f'(3)$  using  $h = 0.02$ .

(b) Find the exact value for  $f'(3)$ .

(c) Give the equation of the tangent line to  $f(x)$  at  $x = 3$ .

(d) Use the tangent line to estimate  $f(2.98)$ .

3. (3 points each) Let  $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

(a) List all critical points for  $f(x)$ .

(b) Use the first derivative test to decide which critical points are local maxima, local minima or inflection points of  $f(x)$ .

(c) Use the second derivative test to decide which critical points are local maxima, local minima or inflection points of  $f(x)$ .

(d) List the global maxima and minima of  $f(x)$  on the interval  $[-1,3]$

4. (3 points each) Let  $f(x) = 12x^2 + 5$

a) Estimate  $\int_1^5 f(x)dx$ . Using LHS and RHS and a width of the interval  $\Delta x = 2$

b) Give two possible antiderivatives for  $f(x)$ .

c) Give the exact value for  $\int_1^5 f(x)dx$ .

d) Give the average value of  $f(x)$  on the interval  $[1,5]$ .

5. (3 points each) A manufacturer produces bolts of a fabric with a fixed width. The quantity  $q$  of this fabric (measured in yards) that is sold is a function of the selling price  $p$  (in dollars per yard), so we can write  $q = f(p)$ . Then the total revenue earned with selling price is  $R(p) = pf(p)$ .

(a)  $f(20) = 10,000$  Means that 20 yards of fabric cost \$10,000.

**TRUE**                      **FALSE**

(b)  $f'(p)$  is measured in  $\frac{\text{yards}}{\$ \text{ per yard}}$ .

**TRUE**                      **FALSE**

(c)  $f'(20) = -350$  means that if we increase the price of the yard by one dollar, we will sell 350 yards less, provided that the ratio remains constant from  $p = 20$  to  $p = 21$

**TRUE**                      **FALSE**

(d)  $R'(p) = f(p) + pf'(p)$

**TRUE**                      **FALSE**

6. (6 points) Make the graph of a function with the following properties:

- $f(x)$  is decreasing for  $0 < x < 3$  and for  $x > 5$
- $f(x)$  is increasing for  $3 < x < 5$
- $f(x)$  is concave down for  $1 < x < 2$  and for  $x > 4$
- $f(x)$  is concave up for  $0 < x < 1$  and for  $2 < x < 4$

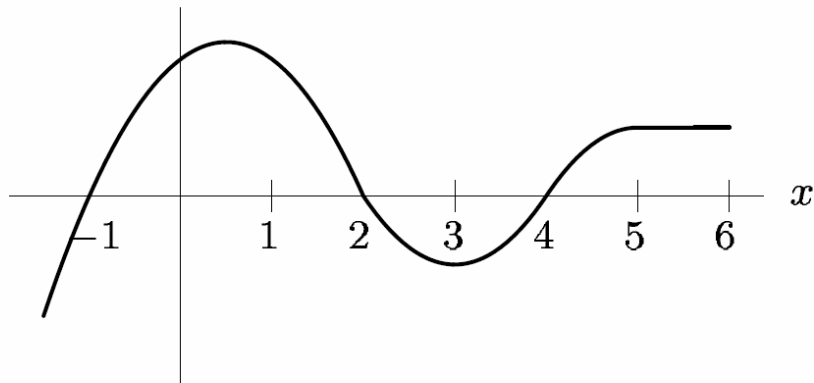
7. (3 points each) Use the rules for differentiation to find the derivative of each of the given functions. Do not simplify.

(b)  $g(x) = \sqrt{\ln(x)}$

(c)  $h(x) = x^e + e^x + e + \pi^x + x^\pi + \pi$

(d)  $i(x) = \frac{\cos x}{\sin x}$

7. (3 points each) The graph below is the graph of  $f'(x)$  (The derivative function of  $f(x)$ ).



- a) List all critical points of  $f(x)$  on  $[-2,5]$ .
- b) Decide if the critical points of  $f(x)$  on  $[-2,5]$  are local maxima, local minima or inflection points.
- c) List all inflection points of  $f(x)$  on  $[-2,5]$ .
- d)  $f(2) = -1$ , and the area between  $f'(x)$  and the x-axis in the interval  $[2,4]$  is 6. Calculate  $f(4)$  using the Fundamental theorem of Calculus.

9. (3 points each) Evaluate each of the following definite or indefinite integrals.

$$(a) \int \left( \frac{x^3}{9} + \frac{4}{x} + \frac{6}{x^3} \right) dx$$

$$(b) \int \frac{4x}{3x^2 + 7} dx$$

$$(c) \int x^2 e^{x^3} dx$$

$$(d) \int_{-1}^1 (x-2)^3 dx$$

10. (7 points) Find the area of the region enclosed by the curves  $y = x$  and  $y = x^2$

