

FINAL EXAM

FALL 2006

December 12, 2006

Show your work, the right solution without explanation is useless. Be clean and organized, it is your responsibility to make yourself understood. No graphic calculators. If you did not bring your calculator, you will have to do the exam without it, sharing calculators is not allowed. Good luck!!!

Question	Points	Out of
1		10
2		10
3		12
4		13
5		12
6		13
7		10
8		10
9		10
Total		100

1. (10 points) The graph of Fahrenheit temperature, $^{\circ}F$, as a function of Celsius temperature, $^{\circ}C$, is a line. You know that $212^{\circ}F$ and $100^{\circ}C$ both represent the temperature at which water boils. Similarly, $32^{\circ}F$ and $0^{\circ}C$ both represent water's freezing point.
- What is the slope of the graph?

Answer: _____

- What is the equation of the line?

Answer: _____

- Use the equation to find what Fahrenheit temperature corresponds to $20^{\circ}C$.

Answer: _____

- What temperature is the same number of degrees in both Celsius and Fahrenheit?

Answer: _____

2. (10 points) The weight, W , in lbs, of a child is a function of its age, a , in years, so $W = f(a)$. Decide if the following statements are True or False. Circle one.
- a. The units of $f'(a)$ are $\frac{\text{lbs}}{\text{year}}$.
TRUE **FALSE**
- b. If $f'(8) = 4$, we can say that a four year old child will increase his weight in 8 lbs. in a year, provided that the ratio stays constant during the fourth year.
TRUE **FALSE**
- c. We expect the weight of a child to increase with time, and therefore we expect $f'(a)$ to be increasing.
TRUE **FALSE**
- d. We expect the weight of a child to increase with time, and therefore we expect $f'(a)$ to be positive.
TRUE **FALSE**
- e. The quantity $\int_1^2 f'(a) da$ represents the total amount of lbs gained by a child the second year.
TRUE **FALSE**
3. (12 points) Use the rules for differentiation to find the derivative of each of the given functions. Do not simplify.

a. $f(x) = 4\sqrt[3]{x^2} - \frac{1}{x^7}$

Answer: _____

b. $g(x) = 5^{\ln x}$

Answer: _____

c. $h(x) = \sqrt{x} \cos(x)$

Answer: _____

d. $i(x) = \frac{3x-2}{x^2+1}$

Answer: _____

4. Let $f(x) = x^3 - 6x^2 + 5$

- a. (3 points) Find the first derivative function $f'(x)$, and use it to determine the critical points of $f(x)$.

Answer: _____

b. (3 points) Determine where $f(x)$ is increasing and decreasing.

Answer: _____

c. (3 points) Find all local maxima and minima.

Answer: _____

d. (4 points) Find all inflection points.

Answer: _____

5. (12 points) Evaluate each of the following definite or indefinite integrals.

a. $f(x) = \int x^3 + \frac{1}{x^7} dx.$

Answer: _____

b. $g(x) = \int \frac{6}{x-2} dx.$

Answer: _____

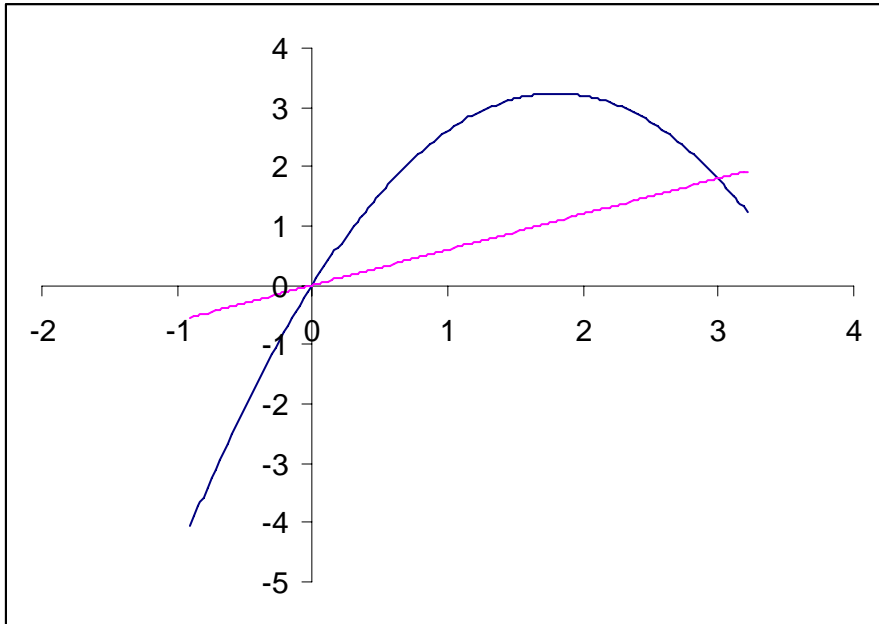
c. $h(x) = \int 6x \sin(3x^2) dx.$

Answer: _____

d. $i(x) = \int_1^2 (3x-1)^2 dx.$

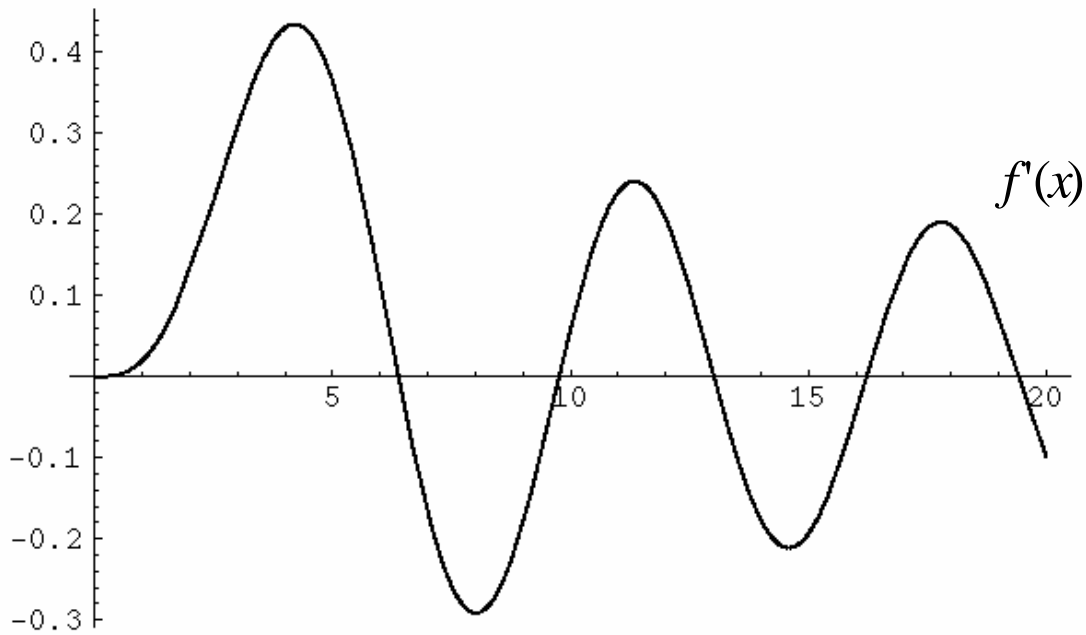
Answer: _____

6. (13 points) Graphs of $f(x) = 0.6x$ and $g(x) = -x^2 + 3.6x$ are given below. Use a definite integral to calculate the area enclosed by the graphs of these two functions.



Answer: _____

7. The graph of the **derivative** function $f'(x)$ is given below.



- (2 points) Mark in the graph the critical points of $f(x)$.
- (2 points) Mark in the graph the intervals in which $f(x)$ is increasing or decreasing.
- (3 points) Mark in the graph all local maxima and minima for the function $f(x)$.
- (3 points) Is $f(4)$ bigger than $f(5)$? Explain.

Answer: _____

8. (10 points) Let $f(x) = x^3 + \ln x$
- What is the tangent line of $f(x)$ at $x = 1$?

Answer: _____

- Use the tangent line to estimate $f(0.9)$.

Answer: _____

9. (10 points) Let $f(x) = x^2 + \frac{\sin(\pi x)}{\pi}$
- Find two antiderivatives of $f(x)$.

Answer: _____

- What is the average value of $f(x)$ in the interval $[1,2]$.

Answer: _____