

Math 77
Midterm 1

Name _____

Spring 2007

Show your work, the right solution without explanation is useless. Be clean and organized, it is your responsibility to make yourself understood. No graphic calculators. If you did not bring your calculator, you will have to do the exam without it, sharing calculators is not allowed. Good luck!!!

1. A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 15% per hour.
 - (a) Write a formula for the amount, A mg, of caffeine in the body t hours after drinking a cup of coffee.

Answer: _____

- (b) Graph the function from part (a).

Answer: _____

- (c) Use logarithms to find the half-life of caffeine.

Answer: _____

2. Decide if the following functions could be linear, exponential or neither. If linear or exponential give the possible formula.

x	f(x)	g(x)	h(x)
0	2	2	2
1	3	3	3
2	4	4.5	4
3	7	6.75	5
4	10	10.125	6

Answer: _____

3. Solve using logs:

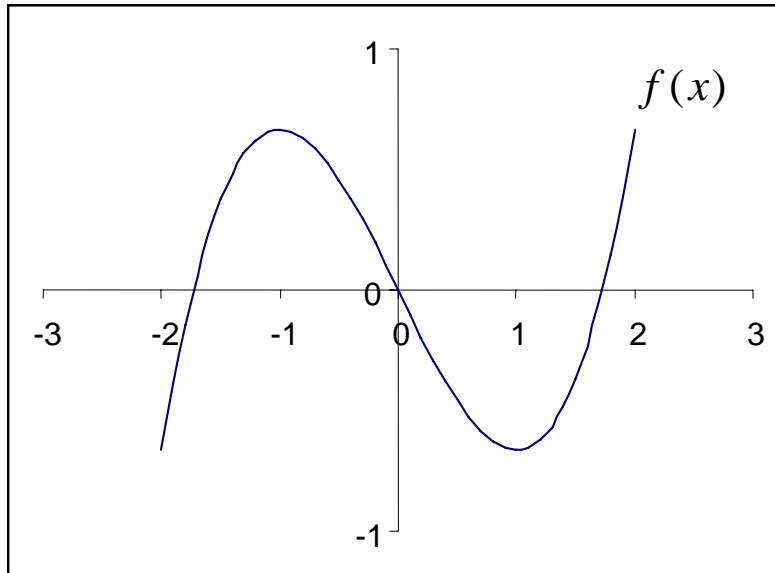
(a) $5^x = 8$

Answer: _____

(b) $e^{7x} = 8e^{2x}$

Answer: _____

4. True or False? Circle one.



(a) Since $f(x)$ is increasing near $x = -1.5$, then $f'(-1.5)$ is negative.

TRUE **FALSE**

(b) Since $f(x)$ attains a local maximum at $x = -1$, then $f'(-1) = 0$.

TRUE **FALSE**

(c) Since $f(x)$ is concave down at $x = 0.5$, then $f'(x)$ is increasing near $x = 0.5$.

TRUE **FALSE**

(d) Since $f(x)$ attains a local maximum at $x = -1$, then $f''(-1) = 0$.

TRUE **FALSE**

5. Let $f(t) = 8t^2 - 2t$:

(a) What is the change in $f(t)$ between $t = 0$ and $t = 2$?

Answer: _____

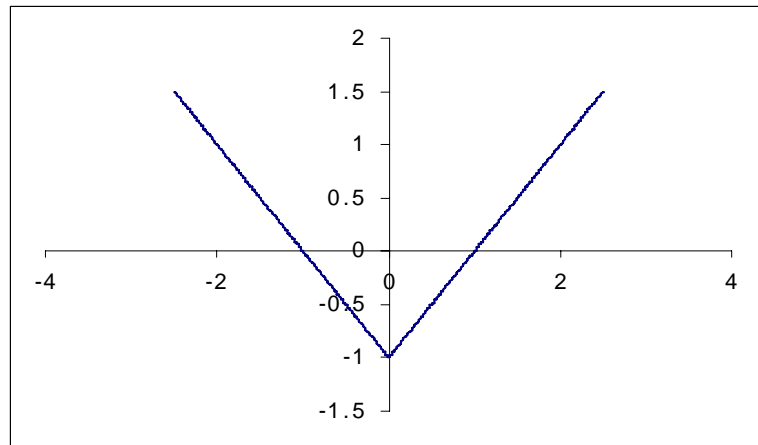
(b) What is the average rate of change in $f(t)$ between $t = 0$ and $t = 2$?

Answer: _____

(c) Give an estimation for $f'(0)$, using $h = 0.07$

Answer: _____

6. Below is the graph of a function $f(x)$. Sketch the graph of its derivative $f'(x)$ on the same axes:



7. Let $f(T)$ be the time, in minutes, that it takes for an oven to heat up to $T^\circ F$
(a) What are the units of $f'(T)$?

Answer: _____

- (b) What is the sign of $f'(T)$? Why? (Here if you answer correctly, but do not give a good explanation, you will not get any point).

Answer: _____

- (c) What is the meaning of $f(300) = 10$?

Answer: _____

- (d) What is the meaning of $f'(300) = 0.1$?

Answer: _____

8. In each case graph a function with the given properties:

- (a) A function whose first and second derivatives are everywhere negative.

- (b) A function whose first derivative is everywhere positive but whose second derivative is everywhere negative.

- (c) A function whose first derivative is constant and negative.

(d) A function whose first derivative is positive for $x \in (-\infty, 0)$ and negative for $x \in (0, \infty)$, and it is everywhere concave down.

9. A population is growing according to the function $P(t) = 200(1.075)^t$, where $P(t)$ is the population at year t .
- (a) What is the initial population?

Answer: _____

- (b) What is the annual growth rate?

Answer: _____

- (c) What is the population in year 10?

Answer: _____

- (d) How many years will it take for the population to reach 1055?

Answer: _____

10. Consider the function $f(x) = 4x^2 - x + 3$

(a) Estimate $f'(2)$ using $h = 0.03$.

Answer: _____

(b) Give the equation of the tangent line at $x = 2$. (Assume that $f'(2) = 15$)

Answer: _____

(c) Estimate $f(1.9)$ using the tangent line

Answer: _____