

**Math 86 — Spring 2004 — Final Exam**  
**Department of Mathematics**  
**Temple University**

May 6, 2004

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

This exam consists of 9 questions. Show all your work. **No work, no credit.** Good Luck!

Question	Points	Out of
1		36
2		6
3		16
4		16
5		6
6		8
7		6
8		6
9		10
Total		110

36 points

1. Evaluate

(a)  $\int_0^1 x^3 \sqrt{x^4 + 1} dx$

(b)  $\int x^2 \sin x dx$

(c)  $\int \cos^4 x \sin^3 x \, dx$

(d)  $\int_0^1 \sqrt{4-x^2} \, dx$

(e)  $\int (1 + \tan^2 x) \sec^2 x \, dx$

(f)  $\int \frac{5x - 1}{x(x^2 + 1)} \, dx$

6 points

2. Determine whether the integral  $\int_0^{\infty} xe^{-2x} dx$  converges or diverges. If it converges, find its value.

16 points

3. Let  $R$  be the region bounded by the parabolas  $y = 1 + x^2$  and  $y = 3 - x^2$ .

(a) Sketch the region  $R$  and find the area of region  $R$ .

(b) Find the volume generated when region  $R$  is rotated about the  $x$ -axis.

16 points

4. Determine whether the series below converge or diverge. Indicate which test was used to show convergence or divergence.

(a) 
$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{n + 3^n}{2^n}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{3n + 1}$$

(d) 
$$\sum_{n=1}^{\infty} \frac{5^n}{n!}$$

6 points 5. Determine whether the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{\sqrt{n^4 + 1}}$$

converges absolutely, converges conditionally or diverges.

8 points 6. Find the radius and the interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2 4^n}$

6 points 7. Find the power series expansion in powers of  $x$  for  $f(x) = \frac{\sin(3x)}{x}$ .

6 points 8. Write  $\int x \ln(1 + x^2) dx$  as a power series in powers of  $x$ .

10 points

9. Let  $f(x) = \int_0^x \sqrt[3]{1-t^2} dt$ ,  $0 \leq x \leq 2$ .

(a) What are  $f(0)$  and  $f'(0)$ ?

(b) Is  $f(1)$  positive or negative? Explain why.

(c) On what interval is  $f(x)$  increasing? Explain why.

(d) Is  $f(x)$  concave up or concave down on  $[0, 2]$ ? Explain why.