

**Math 86 — Fall 2003 — Final Exam**  
**Department of Mathematics**  
**Temple University**

December 9, 2003

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

This exam consists of 9 questions. Show all your work. **No work, no credit.** Good Luck!

Question	Points	Out of
1		36
2		6
3		16
4		16
5		6
6		8
7		6
8		6
9		10
Total		110

36 points

1. Evaluate

(a)  $\int_{-1}^3 \sqrt{2x+3} dx$

(b)  $\int x^2 \ln x dx$

(c)  $\int \sin^2 x \cos^3 x \, dx$

(d)  $\int \frac{dx}{\sqrt{9-x^2}}$

(e)  $\int_0^{\pi/4} \frac{\sec^2 x}{1 + \tan x} dx$

(f)  $\int \frac{5x^2 + 6x + 2}{x(x + 1)^2} dx$

6 points

2. Determine whether the integral  $\int_0^{\infty} xe^x dx$  converges or diverges. If it converges, find its value.

16 points

3. Let  $R$  be the region between the parabola  $y = 4x - x^2$  and the line  $y = x$ .

(a) Find the area of  $R$ .

(b) What is the volume of the solid obtained by rotating  $R$  about the  $x$ -axis?

16 points

4. Determine whether the series below converge or diverge. Indicate which test was used to show convergence or divergence.

(a) 
$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{1+n^2}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{5}{n3^n}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{1+n+n^2}{\sqrt{1+n^2+n^6}}$$

(d) 
$$\sum_{n=1}^{\infty} (-1)^n \frac{3n^2}{4n^2+7}$$

6 points

5. Determine whether the series  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2 + 1}$  converges absolutely, converges conditionally or diverges.

8 points

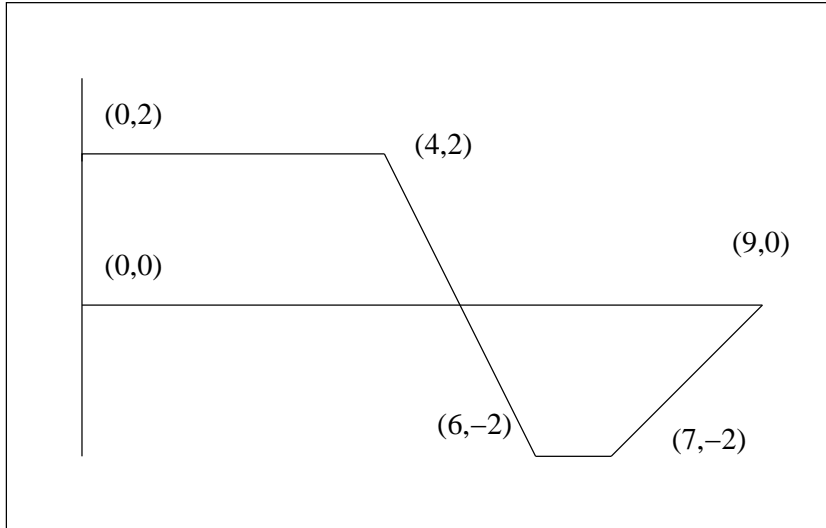
6. Find the radius and the interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{n^2}$

6 points 7. Write  $f(x) = x \ln(1 - x^2)$  as a power series.

6 points 8. Write  $\int \frac{\sin x}{x} dx$  as a power series.

10 points

9. Let  $f(x)$  be the function whose graph is



and let  $g(x) = \int_0^x f(t) dt, 0 \leq x \leq 9$ .

- (a) What are  $g'(2)$  and  $g''(2)$ ? Explain why.
  
  
  
  
  
  
  
  
  
  
- (b) What is  $g(9)$ ? Explain why.
  
  
  
  
  
  
  
  
  
  
- (c) On what interval is  $g(x)$  increasing?
  
  
  
  
  
  
  
  
  
  
- (d) Where does  $g$  have a maximum value and what is that value? Explain why.

Answer:  $g$  has a maximum value of ..... at  $x = \dots\dots\dots$  .