

Math 76 — Spring 2004 — Final Exam
Department of Mathematics
Temple University

May 6, 2004

Name: _____

Instructor: _____

This exam consists of 8 questions. Show all your work. **No work, no credit.** Good Luck!

Question	Points	Out of
1		36
2		14
3		10
4		8
5		10
6		10
7		12
8		10
Total		110

36 points

1. Evaluate

(a) $\int_0^3 \frac{1}{\sqrt{1+x}} dx$

(b) $\int x^2 e^{x^3} dx$

(c) $\int x \cos(2x) dx$

(d) $\int_{-1}^0 \frac{dx}{x^2 + 2x + 2}$

(e) $\int_0^{\pi/2} \cos x \sqrt{1 + \sin x} \, dx$

(f) $\int \frac{2x - 5}{(x - 2)(x + 1)} \, dx$

14 points

2. Compute the following improper integrals:

(a) $\int_{-\infty}^0 \frac{e^x}{1+e^x} dx$

(b) $\int_2^6 \frac{dx}{\sqrt{x-2}}$

10 points

3. Sketch the region bounded by $y = \sin(2x)$ and $y = 2 \sin x$ for $0 \leq x \leq \pi$ and find its area.

8 points

4. The region bounded by $y = 3x^4$, $y = 0$, $x = 0$, $x = 2$ is rotated around the x -axis. Find its volume.

10 points

5. Use 4 steps of Euler's method to determine an approximate solution y at $x = 2$ for the differential equation $\frac{dy}{dx} = -xy$ with $y(0) = 2$. Give your answer by filling in the table below.

x	0	0.5	1	1.5	2
y	2				
$\frac{dy}{dx}$					

10 points

6. Solve the differential equation $\frac{dy}{dt} = y + yt^3$ subject to the initial condition $y(0) = 2$.

12 points

7. After a drug is fully absorbed, the amount of the drug in the body decreases at a rate proportional to the amount left in the body.

(a) Write a differential equation for the amount of the drug t hours after it is fully absorbed.

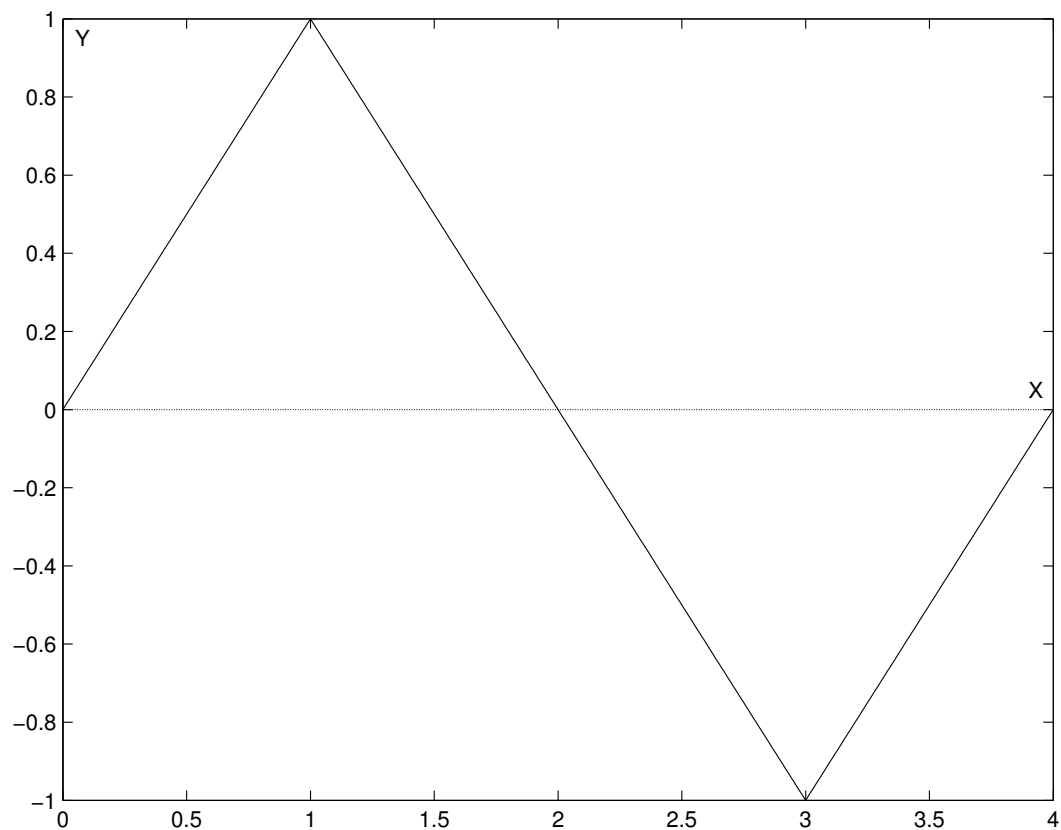
(b) Find the general solutions of the differential equation.

(c) Suppose the half-life of the drug in the body is 3 hours, find the decay constant.

(d) Suppose 20 mg of a drug with the half-life of 3 hours is injected in the body. Find the amount of the drug in the body 5 hours after injection.

10 points

8. The graph of the function $y = f(x)$ is given in the figure below.



Set

$$g(x) = \int_0^x f(t) dt.$$

(a) Find $g(2)$ and $g(4)$

(b) Find $g'(2)$.

(c) For which value of x in the interval $[0, 4]$ does the function $g(x)$ take on its maximum value? Give an explanation for your answer.