

1. (18 points) Find

(a)  $\int \frac{1}{5 + 3x} dx$

(b)  $\int \cos^3(2x) \sin(2x) dx$

(c)  $\int x^{-2} \ln x dx$

2. (18 points) Evaluate

(a)  $\int_0^{\frac{1}{2}} \frac{1}{1 + 4x^2} dx$

(b)  $\int e^{-2x} \sqrt{1 + e^{-2x}} dx$

(c)  $\int \frac{x}{(x - 1)(x + 2)} dx$

3. (12 points) Compute the following improper integrals:

(a)  $\int_2^{\infty} \frac{3}{x^5} dx$

(b)  $\int_1^5 \frac{1}{(5 - x)^{\frac{4}{3}}} dx.$

4. (8 points) Suppose a rock is thrown upward from a 36 foot high bridge with the initial velocity of 64 ft/sec.

(a) Find the velocity  $v(t)$  and the height  $h(t)$  of the rock  $t$  seconds after the throw. Note that the acceleration of gravity  $g = 32 \text{ ft/sec}^2$ .

(b) When will the rock hit the ground?

(c) What is the velocity of the rock when it hits the ground?

5. (8 points) Sketch the region, bounded by the curves  $y = x^2$  and  $y = 2x + 3$  and find its area.

6. (8 points) The region bounded by  $y = x^3$ ,  $x = 0$  and  $y = 1$  is rotated around the  $x$ -axis. Find the volume of the solid of revolution.

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7. (8 points) Use the Euler's method to approximate  $y$  at  $x = 2$  in 4 steps where  $y(x)$  is the solution to the differential equation  $\frac{dy}{dx} = y + 2x$  with an initial condition  $y(0) = -1$ .

8. (10 points) Solve the differential equation  $\frac{dy}{dx} = e^{-y}(2 + 3x^2)$  subject to the initial condition  $y(-2) = 0$ .

9. (10 points) Assume that the spread of a disease causes the rate of growth of the infected population to be proportional to the size of the population already infected. Let  $P(t)$  be the infected population at time  $t$ .

(a) Write a differential equation satisfied by  $P(t)$  as a function of time  $t$

(b) Find the general solution to the above equation.

(c) Find the solution to the above equation given that initially there were 2000 infected people and one month later the number of infected people was 3000.

(d) Determine the number of infected people 6 months after the start of the epidemic.