Math 75 Instructor & Section: ________________ **PRINT Your Name:** ____________________

MATH 75 FINAL EXAM **SPRING 2004** **MAY 6, 2004**

This exam consists of 12 questions.

SHOW ALL YOUR WORK! NO WORK, NO CREDIT.

It is recommended that you draw a box around your final answer to each problem.

Question	Points	Out of
1		20
2		6
3		6
4		4
5		4
6		8
7		10
8		10
9		10
10		6
11		8
12		8
EXTRA CREDIT		5
TOTAL		

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1. Compute the derivative of each of the following expressions. **SHOW YOUR STEPS and draw a box around your final answer. DO NOT SIMPLIFY!**

a) $\mathbf{f(x)} = \mathbf{sin}^2\mathbf{(x)} + \frac{2}{\sqrt{\mathbf{x}}}$

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b) $\mathbf{g(x)} = \mathbf{xe}^{\mathbf{x}^2}$

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c) $\mathbf{y} = \left(1 - \mathbf{x}^3\right)^{-1/3} + \mathbf{ln}\left(1 - \mathbf{x}\right)$

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d) $\mathbf{w} = \frac{\mathbf{cos(}2\mathbf{x)}}{1 + 2\mathbf{x}}$

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#2. Given $\mathbf{f(1)} = 4, \mathbf{f'(1)} = 3, \mathbf{g(1)} = 2$, **and** $\mathbf{g'(1)} = -1$. Draw a box around your answers.

a) Find the derivative of $\mathbf{y} = \sqrt{\mathbf{f(x)}}$ at x = 1.

b) Find the derivative of $\mathbf{y} = \mathbf{g(x)\,f(x)}$ at x = 1.

#3. Given $\mathbf{y^2 - 4x + 7y = 14}$.

Find $\frac{\mathbf{dy}}{\mathbf{dx}}$. (Show your steps and draw a box around your final answer.)

4. Determine the equation of the tangent line to $\mathbf{f(x)} = \sqrt{\mathbf{x}}$ at x = 9.

____

(answer)

5. Given that $\mathbf{f(x) = 3\ln(x)}$. Use local linearization at x = 1 to estimate **f(0.9).**

____

(answer)

6. A tennis ball is thrown straight up from the edge of a 128 ft. tall building so that when it passes the top of the building on its way down it continues towards the ground. The height of the tennis ball in feet after t seconds is given by $\mathbf{h(t) = -16t^2 + 32t + 128}$.

a) When will the tennis ball change from rising to falling?

____
(answer)

(b) When will the velocity of the tennis ball be **-64 ft** per second?

____
(answer)

(c) When will the tennis ball strike the ground? (Give the exact answer.)

____
(answer)

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7. Compute the following limits. SHOW YOUR WORK.

a) $\lim_{x \to \infty} \frac{x^2 + 2}{1 + 3x + x^3}$

____
(answer)

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b) $\lim_{x \to 0} \frac{e^x - 1}{x}$

____
(answer)

---

c) Let $f(x) = 3x^2$. Compute $f'(3)$ using the definition $\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$. **(Your answer must be completely justified by your work!)**

____
(answer)

8. Let $\mathbf{f(x)} = \begin{cases} \mathbf{cos(x)} + 1 & \textbf{for } \mathbf{x} < 0 \\ 3\mathbf{x} + 2 & \textbf{for } 0 \le \mathbf{x} \le 2 \end{cases}$

a) What is $\lim_{x \to 0^+} f(x)$? Answer ➔ ________________________

b) Determine $\lim_{x \to 0^-} f(x)$. Answer ➔ ___________________

c) Determine $\lim_{x \to 0} f(x)$ Explain your answer. Answer ➔ _____________________

EXPLANATION: ____

____

d) Is **f(x)** continuous at **x = 0**? Explain your answer. Answer ➔ ___________________

EXPLANATION: ____

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e) Is **f(x)** differentiable at **x = 0**? Explain your answer. Answer ➔ ___________________

EXPLANATION: ____

____

9. Given the function $\mathbf{f(x)} = \mathbf{x}^3 - 3\mathbf{x}^2 - 9\mathbf{x} + 10$ and $\mathbf{-2} \le \mathbf{x} \le \mathbf{6}$. Determine the items in parts (a) – (d) for **f(x).** (For any points requested, if it exists you must specify both **x** and **y** coordinates, otherwise write "does not exist". **SHOW ALL WORK!**

(a) Find the first and second derivative of **f(x)**.

First Derivative ________________ Second Derivative ___________________

(b) Find the critical points and determine those that are local max and those that are local min. **(Specify both x and y coordinates.) You must justify your answer.**

Local Maximum Point(s) __________________ Local Minimum Point(s) __________________

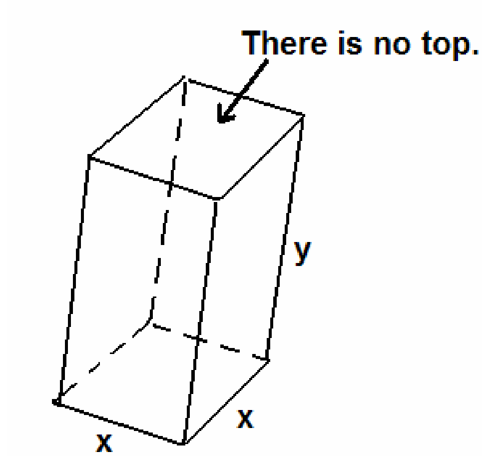
(c) Find any inflection points. **(Specify both x and y coordinates.)**

Inflection point(s) __________________________________

(d) Determine the global max and global min. **(Specify both x and y coordinates.)**

Global Maximum Point(s) ______________ Global Minimum Point(s) ________________

10. A box with a square base and open top has volume 32 cubic meters. Find the dimensions of the box that minimizes the amount of material needed to construct the sides and bottom of the box.



Dimensions: x = _________________ y = _________________

11. A sketch of function y = f(x) is given in the figure below.

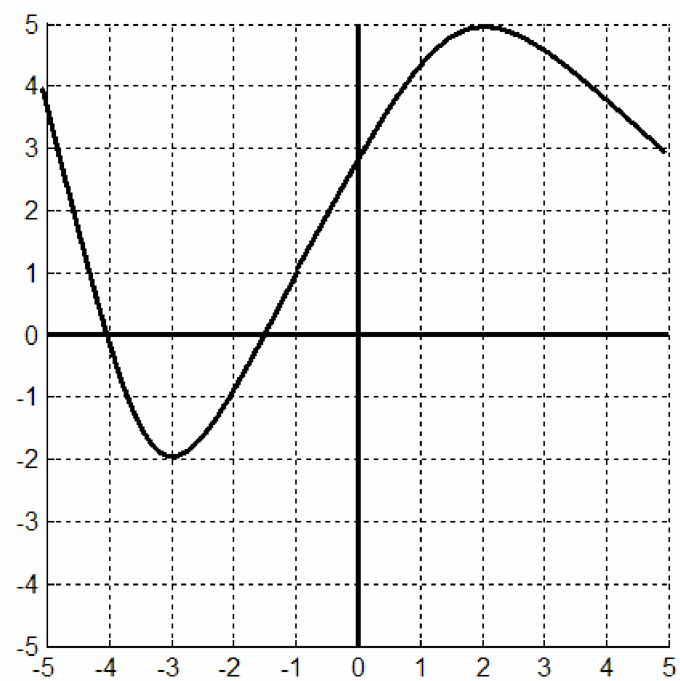
<u>Sketch of y = f(x).</u>

(a) Determine all intervals where **f '(x) > 0.**

____

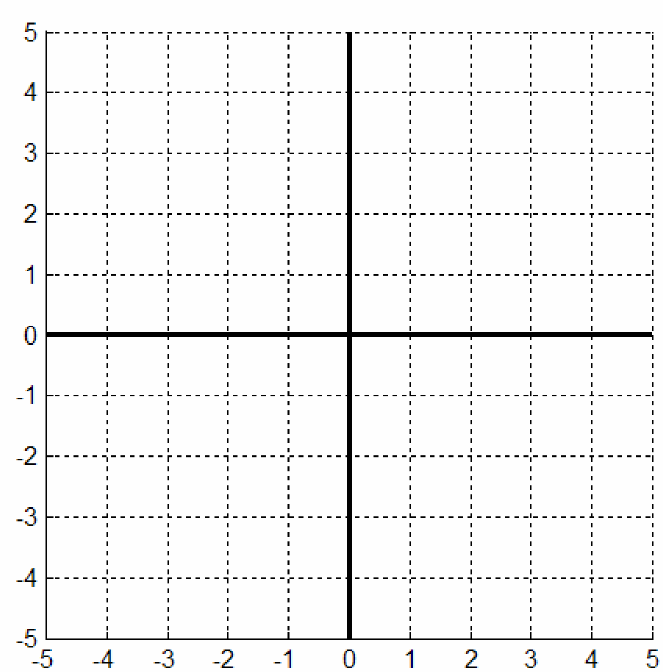
(b) Determine all intervals where **f '(x) < 0.**

____



(c) Make a sketch of **f '(x)** on the grid provided below.

Sketch **f '(x)** below!



(d) On your sketch of **f '(x)** place the letter **P** on the x-axis that indicates the approximate location of the x-coordinate of the inflection point of the **<u>original</u> function y = f(x)**.

12. The temperature, **H**, in degrees Fahrenheit **(ºF)**, of a can of soda that is put into a refrigerator to cool is given as a function of time, t, in hours, by

$$\mathbf{H(t) = 40 + 20e^{-t}}$$

(a) What is the temperature of the soda when it is initially put into the refrigerator?

Initial temperature = ___________________________ (include units)

(b) What is the temperature one hour later? (Include units)

Temperature after 1 hour = ___________________________ (include units)

(c) Find the rate of change of the temperature one hour after the soda is placed in the refrigerator?

Rate of Temperature Change
after 1 hour = ___________________________ (include units)

(d) When will the temperature of the can of soda be 50ºF?

Time when temperature is 50ºF = ______________________________

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EXTRA CREDIT!

On the line provided PRINT the appropriate response TRUE or FALSE.

__________ 1. If $f(x)$ is increasing, then $f'(x)$ is increasing.

__________ 2. If $f''(x) > 0$, then $f'(x)$ is increasing.

__________ 3. If a function is continuous at x = a, then it is differentiable at x = a.

__________ 4. If $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{g'(x)}{h'(x)}$.

__________ 5. If $f'(p) = 0$, then f has a local maximum or local minimum at $x = p$.