Math 75 Instructor: _______________ **PRINT Your Name: ___________________**

MATH 75 FINAL EXAM **Fall 2004** **DEC 16, 2003**

This exam consists of 11 questions.

SHOW ALL YOUR WORK! NO WORK, NO CREDIT.

It is recommended that you draw a box around your final answer to each problem.

Question	Points	Out of
1		20
2		6
3		6
4		8
5		8
6		10
7		8
8		10
9		10
10		8
11		6
EXTRA CREDIT		5
TOTAL		

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1. Compute the derivative of each of the following expressions. **<u>SHOW YOUR STEPS and draw a box around your final answer. DO NOT SIMPLIFY!</u>**

a) $\mathbf{f(x) = \sin^3(x) + \sin(x^3)}$

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b) $\mathbf{g(x) = \ln\left(\frac{e^{5x}}{x^2 - 2}\right)}$

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c) $\mathbf{y = x^2 \arctan(6x)}$

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d) $\mathbf{w = \sqrt[5]{1 + \tan(7x)}}$

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#2. Given $\mathbf{f(3)} = 2, \mathbf{f'(3)} = -8,$, **and** $\mathbf{f'(2)} = 5$. Draw a box around your answers.

a) Find the derivative of $\mathbf{y} = \dfrac{1}{\mathbf{f(x)}}$ at x = 3.

b) Find the derivative of $\mathbf{y} = \mathbf{f}\left(\mathbf{f(x)}\right)$ at x = 3.

#3. Find the slope of the tangent line to the ellipse $\dfrac{\mathbf{x^2}}{\mathbf{25}} + \dfrac{\mathbf{y^2}}{\mathbf{9}} \mathbf{= 1}$ at $\left(3, \dfrac{12}{5}\right)$. (Show your steps and draw a box around your final answer.)

4. a) Determine the equation of the tangent line to $\mathbf{f(x)} = \dfrac{\mathbf{1}}{\mathbf{1+x}}$ at x = 1.

____
(answer)

b) Given that **g(x) = 2 sin(x)**. Use local linearization (that is, use the tangent line) at x = 0 to estimate **2 sin(0.3)**;

____
(answer)

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5. An orange is thrown straight up from the edge of a 96 ft. tall building so that when it passes the top of the building on its way down it continues towards the ground. The height of the orange after t seconds is given by $\mathbf{h(t) = -16t^2 + 16t + 96}$.

a) When will the orange be at its maximum height above the ground?

____
(answer)

b) What is the velocity of the orange when it passes the top of the building on the way down?

____
(answer)

c) When will the orange strike the ground? (Give the exact answer.)

____
(answer)

6. Compute the following limits. SHOW YOUR WORK.

a) $\lim_{x\to\infty} \frac{x\left(1+x^2\right)}{3x^2}$

____
(answer)

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b) $\lim_{x\to 1} \frac{(x-1)^2}{\ln(x)}$

____
(answer)

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c) Let $f(x) = x^2 + 1$. Compute $f'(2)$ using the definition $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$. **(Your answer must be completely justified by your work!)**

____
(answer)

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7. Let $\mathbf{f(x)} = \begin{cases} \mathbf{x} & \textbf{for x} < 0 \\ 1 & \textbf{for x} = 0 \\ \mathbf{x} - 1 & \textbf{for x} > 0 \end{cases}$

a) What is $\lim\limits_{x \to 0^+} \mathbf{f(x)}$? Answer ➔ ________________________

b) Determine $\lim\limits_{x \to 0^-} \mathbf{f(x)}$. Answer ➔ ___________________

c) Determine $\lim\limits_{x \to 0} \mathbf{f(x)}$ Explain your answer. Answer ➔ _____________________

EXPLANATION: ____

____

d) Is **f(x)** continuous at **x = 0**? Explain your answer. Answer ➔ ___________________

EXPLANATION: ____

____

e) Is **f(x)** differentiable at **x = 0**? Explain your answer. Answer ➔ ___________________

EXPLANATION: ____

____

8. Given the function $\mathbf{f(x)} = 2\mathbf{x}^3 - 3\mathbf{x}^2 - 36\mathbf{x} + 1$, and $-5 \leq x \leq 5$. Determine the following for **f(x)**, if they occur: critical points, global maxima, and global minima. (If they exist you must specify both **x** and **f(x)**, otherwise write "does not exist". **<u>Your final answer must be completely justified by your written work.</u>**) **<u>SHOW ALL WORK!</u>**

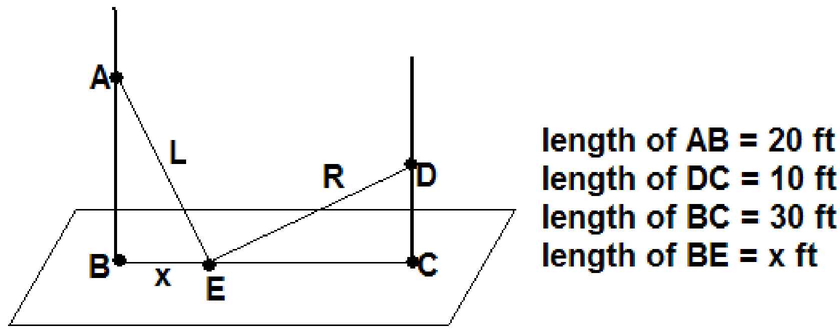
Critical Points (give x- and y-coordinates) ______________________________________

Global Max Point: x = __________ f(x) = _____________

Global Min Point: x = __________ f(x) = ______________

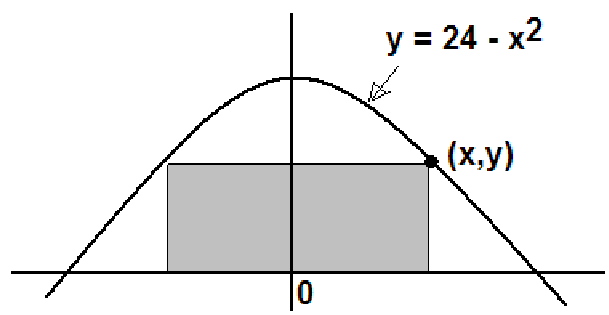
9. (a) Two flagpoles mounted on a roof are 30 feet apart. They are to be anchored by wires of length L and R as shown, such that the total length of wire used is as small as possible.
Construct the function to be optimized in terms of the single variable x.

(DO NOT SOLVE THE OPTIMIZATION PROBLEM.)



Function to optimize: ____

(b) A rectangle is to be constructed as shown in the figure. Determine the dimensions of the rectangle of maximum area.
SHOW YOUR WORK!



Dimensions: width =_________ height =_________

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10. A sketch of the graph of the **derivative** of the function **y = f(x)** over the interval [-3,3] is given below.

a) State the intervals over which **f(x)** is increasing.

____

b) State the intervals over which **f(x)** is decreasing.

____

c) State the x-coordinates of any local maxima.

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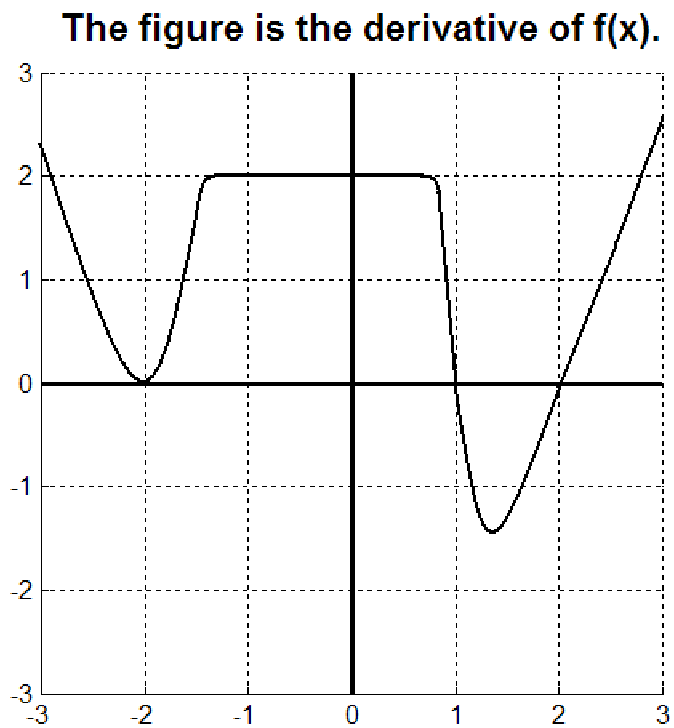

d) State the x-coordinates of any local minima.

____

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11. The temperature, H, in degrees Fahrenheit (ºF), of a can of soda that is put in the refrigerator to cool is given as a function of time, t, in hours, by

$$H = 40 + 30e^{-2t} .$$

a) Find the temperature of the soda at t=0 and at t=2 hours.

Answer at t = 0 ➔ _________________ Answer at t = 2 ➔ _________________

b) Find the rate at which the temperature is changing at t=0 and at t=2 (in ºF/hour).

Answer at t = 0 ➔ _________________ Answer at t = 2 ➔ _________________

c) If the can of soda is left in the refrigerator for a very long time what is the limiting value of its temperature? Explain your answer.

Answer ➔ _________________

Explanation: ____

____

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<u>EXTRA CREDIT PROBLEM</u> (5 points)

<u>Draw a circle around the expressions that are</u> **<u>bounded</u>** (that is, they have both an upper bound and a lower bound) over the accompanying interval.

a) $\mathbf{f(x) = 3\cos(5x)}$ **on** $\mathbf{(-\infty, \infty)}$

b) $\mathbf{g(x) = e^{x}}$ **on** $\mathbf{(-\infty, \infty)}$

c) $\mathbf{h(x) = \frac{1}{x}}$ **on** $\mathbf{[1, 10]}$

d) $\mathbf{p(x) = \tan(x)}$ **on** $\mathbf{[0, \pi/2)}$

e) $\mathbf{w(x) = 2x + 1}$ **on** $\mathbf{[0, \infty)}$