MATH 117 SPRING 2001: MIDTERM UNO

CRISTIAN GURITA

Problem 1. *(4 points)*

(1) *Find the Taylor polynomials of degree 5 at $x = 0$ for $f(x) = \sin x$ and $g(x) = \cos x$.*
(2) *Find out if the series*

$$\sum_{k=1}^{\infty} \frac{1}{(2k+1)^3}$$

is convergent or divergent.

Problem 2. *(4 points)*

(1) *Determine if the series $\frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} \ldots$ is convergent or not, and if it is, find its sum.*
(2) *Find the Taylor series at $x = 0$ for the function $f(x) = \frac{1}{(1+x)^2}$.*

Problem 3. *(4 points)*

(1) *Find $D_t\mathbf{r}(t)$ and $D_t^2\mathbf{r}(t)$ for $\mathbf{r}(t) = (3t+4)^3\mathbf{i} + e^{t^2}\mathbf{j}$.*
(2) *For the plane curve given by $x = e^t \cos t\mathbf{i} + e^t \sin t\mathbf{j}$, find the unit tangent vector $\mathbf{T}$ and the curvature $\kappa(t)$ at the point $t_1 = \frac{\pi}{2}$.*

Problem 4. *(4 points)*

(1) *Find dy/x and d^2y/dx^2 without eliminating the parameter for the curve given by $x = 2t^2, y = \sqrt{5}\,t^3$.*
(2) *If $\mathbf{r}(t) = e^{-t}\mathbf{i} + e^t\mathbf{j}$, find the velocity and acceleration vectors, $\mathbf{v}(t)$ and $\mathbf{a}(t)$ and the speed at time $t = 1$.*

Problem 5. *(4 points)*

(1) *Find the cosine of the angle between the vectors $\mathbf{a} = (1, -3)$ and $\mathbf{b} = (-1, 2)$*
(2) *For what numbers c are the vectors $(c, 6)$ and $(c, -4)$ orthogonal?*

Extra credit. *(2 points)*

(1) *In problem 1.1, what is funny about the two Taylor polynomials? Explain clearly (we talked about this in class).*
(2) *For the motion given by $\mathbf{r} = t^2\mathbf{i} + \frac{1}{3}t^3\mathbf{j}$ find the tangential acceleration a_T and the normal acceleration a_N. (This is extra credit because we didn't finish this example in class).*