RESEARCH PLAN

CRISTIAN GURITA

Department of Mathematics, Temple University

The theory of elliptic boundary value problems is by now well understood, having been developed to a high degree of sophistication. There are two main approaches: First, majorizing *some* norm of a solution u by *some* norms of the data and then using functional analysis arguments. The second approach involves transferring the problem entirely to the boundary of the domain and solving there a problem on a manifold without boundary. The conditions for the solvability of this second problem are known as the Shapiro-Lopatinskiĭ conditions.

One would be hard pressed to cite all the relevant work in this field and it is risky to select just a few. However, the paper of Agmon-Douglis-Nirenberg [1], deriving estimates near the boundary, can be considered to be crucial for the first approach, and Hörmander [3], concerning estimates for systems of pseudo-differential operators on the boundary, can be considered to be essential for the second approach. The original paper of Lopatinskiĭ is translated in [5].

In the problem I investigate the elliptic operator will be replaced by a certain type of nonelliptic operators. A local example in R^4 can be given using four vector fields: $L_1 = x_4\partial_{x_1}$, $L_2 = \partial_{x_2} + x_3\partial x_1$, $L_3 = \partial_{x_3} - x_2\partial_{x_1}$, $L_4 = \partial_{x_4}$. Then $P(x, D) = L_1^2 + L_2^2 + L_3^2 + L_4^2$.

The problem to understand is: What is, in this case, the analogous of the Shapiro-Lopatinski conditions for the "well-posedness" of the boundary value problem? It's clear that some sort of microlocal condition will be needed in addition to the appropriate classical boundary conditions.

The class of operators I am working with have symplectic characteristic set lying over the boundary and are hyppoelliptic with loss of one derivative. The operator P above falls in this class. It satisfies Hörmander's bracket condition, namely, the vector fields $L_1, ..., L_4$ and their first order brackets span the algebra of all vector fields on R^4, hence the operator is hypoelliptic with loss of one derivative (see [4]). This general class of operators I is well understood, beginning with Boutet de Monvel [2] and Sjöstrand [6]. In [7], Taylor sums up the results in this direction, stressing the close connection between the hypoellipticity of the operator P and the non-involutivity of its characteristic set. However, the possibility of setting up boundary value problems with them has not yet been investigated.

The idea is to apply these results, which refer to plain partial differential equations (with no boundary conditions), to the case of nonelliptic boundary value problems. After transferring the boundary value problem to a problem on the boundary only, there should be a close connection between the characteristic set of the pseudodifferential operators obtained this way (the trace of the original partial differential operators on the boundary) and the characteristic set of the pseudodifferential operators giving the boundary conditions. I believe that the microlocal interaction between these sets is the key to understanding this kind of non-elliptic boundary value problems.

The motivation for studying this problem is that these operators arise as variants of the Bergman Laplacian on a strictly pseudoconvex open set in C^n with smooth boundary. As such, it may add to our understanding of the analysis of problems in several complex variables.

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References

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